MATHEMATICS-X

MODULE-1

INDEX

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REAL NUMBERS

1.1 BASIC CONCEPTS AND IMPORTANT RESULTS

(a) Natural numbers : Counting numbers are called natural numbers. We start counting from 1, so 1 is the smallest natural number. A set of natural numbers is denoted by N. Thus,

N = {1, 2, 3, ...up to the numbers as possible as we can count}

(b) Whole numbers : Natural numbers together with zero are called whole numbers. A set of whole numbers is denoted by W. Thus,

W = {0, 1, 2, 3, ...}

All natural numbers are whole numbers but all whole numbers are not natural numbers. Only the difference between whole numbers and natural numbers is the number zero (0).

(c) Integers : All positive and negative natural numbers together with zero are called integers. A set of integers denoted by Z or I. Thus,

Z = {.... – 6, – 5, – 4, –3, – 2, –1, 0, 1, 2, 3, 4, 5, 6 ....}

(d) Rational numbers : A number of the form $\frac{p}{q}$, where p and q are integers and $q \neq 0$ is called a rational number. For example, $\frac{2}{7}$, $\frac{-3}{8}$, 2, 0, etc., are rational number.

The decimal expansion of a rational number is either terminating or non-terminating repeating one.

(e) Irrational numbers : A number which is not a rational number, i.e., which cannot be written in the form $\frac{p}{q}$, p and $q \in$ Z or I and $q \neq 0$, is known as an irrational number. For example,

$\sqrt{2}$, $-\sqrt{3}$, $7\sqrt{5}$, $\frac{1}{\sqrt{2}}$, $1 + \sqrt{6}$, etc., are irrational numbers.

The decimal expansion of an irrational number is non-terminating and non-repeating one.

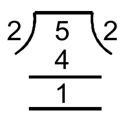
(f) Real numbers : All rational and irrational numbers together make up a collection, called real numbers.

Note : *All natural numbers, integers, rational numbers and irrational numbers are real numbers.*

1.2 EUCLID'S DIVISION ALGORITHM OR EUCLID'S DIVISION LEMMA

For any two given positive integers a and b, there exist unique whole numbers q and r such that:

$a = b \times q + r$, where $0 \leq r < b$



Here, a is called the dividend, b the divisor, q the quotient and r is called the remainder. For example, when we divide 5 by 2, we get 2 as quotient and 1 as remainder

Here $5 = 2 \times 2 + 1$

$\therefore$ Dividend = Divisor × Quotient + Remainder

Properties of Euclid's Division Lemma :

(i) If an integer c be a divisor of each of the two given integers a and b, then we say that c is a common factor of both a and b.

(ii) Let P be a prime number and if a,b, are integers such that $\frac{P}{ab}$, then either $\frac{P}{a}$ or $\frac{P}{b}$ is a factor.

Note : If a prime number divides a product of integers, then it necessarily divides either of the integers.

Obtaining HCF by Euclid Division Lemma :

Let a and b be two positive integers. If $a = b \times q + r, 0 \le r < b$

Then HCF (a, b) = HCF (b, r)

where HCF(a, b) = Common division of a and b and

HCF (b, r) = Common division of b and r.

1.3 FUNDAMENTAL THEOREM OF ARITHMETIC

Every composite number can be expressed as a product of primes and this factorization is unique, apart from the order in which the prime factors occur.

Note : *Any number of the form a^n will end with the digit zero if prime factors of a^n would contain 5 and 2 as a prime factor, where $n \in W$.*

1.4 THEOREMS ON RATIONAL NUMBERS

(i) Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$, where p and q are co-prime, and the prime factorization of q is of the form 2^n5^m, where n, m are non-negative integer.

(ii) Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form 2^n5^m , where n, m are non-negative integers. Then x has a decimal expansion which terminates.

(iii) Let $x = \frac{p}{q}$ be a rational number such that the prime factorization of q is not of the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which is non-terminating repeating (recurring).

Note : $HCF(p, q, r) \times LCM(p, q, r) \neq (p \times q \times r)$, where p, r and q are positive integers.

$$LCM(p, q, r) = \frac{p \times q \times r.HCF(p,q,r)}{HCF(p,q).HCF(q,r).HCF(p,r)}$$

$$HCF(p, q, r) = \frac{p \times q \times r.LCM(p,q,r)}{LCM(p,q).LCM(q,r).LCM(p,r)}$$

SOLVED PROBLEMS

Ex.1 Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$ or $6q + 5$, where q is some integer.

Sol. Let us start with taking a, where a is any positive odd integer. We apply the division algorithm, with a and $b = 6$. Since $0 \le r < 6$, the possible remainders are 0, 1, 2, 3, 4, 5. That is, a can be 6q or $6q + 1$, or $6q + 2$, or $6q + 3$, or $6q + 4$, or $6q + 5$, where q is the quotient. However, since a is odd, we do not consider the cases 6q, $6q + 2$ and $6q + 4$ (since all the three are divisible by 2). Therefore, any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$.

Ex.2 If $a = 10$ and $b = 3$. Then find q and r such that $a = bq + r$, $o \le r < b$.

Sol. Clearly $10 = 3 \times 3 + 1$

$\therefore$ $q = 3$ and $r = 1$ are required numbers.

Ex.3 Use Euclid's division algorithm to find the HCF of 135 and 225.

Sol. We start with the larger integer, that is, 225. Then by Euclid's division algorithm we get:

$$225 = 135 \times 1 + 90$$

Now, consider the divisor 135 and the remainder 90 and apply the division algorithm again, we get :

$$135 = 90 \times 1 + 45$$

Now, consider the divisor 90 and the remainder 45 and apply the division algorithm again, we get.

$$90 = 45 \times 2 + 0$$

Notice that the remainder has become zero and we cannot proceed any further. The HCF of 90 and 45 is 45 and **we claim** that the HCF of 225 and 135 is also **45.**

Ex.4 An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Sol. Hint : Find HCF of 616 & 32

Ex.5 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or $3m + 1$ for some integer m.

Sol. Let a be any odd positive integer. We apply the division lemma with a and $b = 3$.

Since $0 \le r < 3$, the possible remainders are 0, 1 and 2. That is, a can be 3q, or $3q + 1$, or $3q + 2$, where q is the quotient.

Now, $(3q)^2 = 9q^2$

which can be written in the form 3m, since 9 is divisible by 3.

Again, $(3q + 1)^2 = 9q^2 + 6q + 1 = 3\,(3q^2 + 2q) + 1$

which can be written in the form $3m + 1$ since $9q^2 + 6q$, i.e., $3(3q^2 + 2q)$ is divisible by 3.

Lastly, $(3q + 2)^2 = 9q^2 + 12q + 4 = (9q^2 + 12q + 3) + 1 = 3(3q^2 + 4q + 1) + 1$

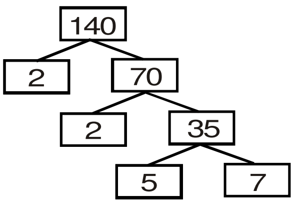
which can be written in the form $3m + 1$, since $9q^2 + 12q + 3$, i.e., $3(3q^2 + 4q + 1)$ is divisible by 3.

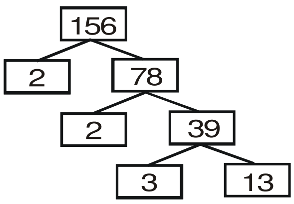
Therefore, the square of any positive integer is either of the form 3m or $3m + 1$ for some integer m.

Ex.6 Express each number as product of its prime factors :

(i) 140 (ii) 156 (iii) 3825 (iv) 5005 (v) 7429

Sol. (i) 140 (ii) 156





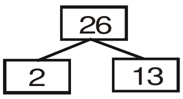
So, $140 = 2 \times 2 \times 5 \times 7 = 2^2 \times 5 \times 7$ So, $156 = 2 \times 2 \times 3 \times 13 = 2^2 \times 3 \times 13$

(Rest Try your self)

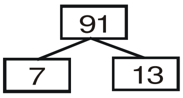
Ex.7 Find the LCM and HCF of the following pairs of integers and verify that LCM × HCF = product of two numbers.

(i) 26 and 91 (ii) 510 and 92 (iii) 336 and 54

Sol. (i) 26 and 91



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore, LCM (26, 91) $= 2 \times 7 \times 13 = 182$

HCF (26, 91) $= 13$

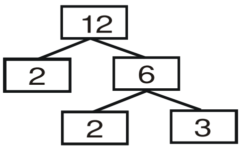
Verification LCM × HCF $= 182 \times 13 = 2366$ and $26 \times 91 = 2366$

i.e., LCM × HCF = product of two numbers. **(Rest Try your self)**

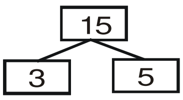
Ex.8 Find the LCM and HCF of the following integers by applying the prime factorisation method.

(i) 12, 15 and 21 (ii) 17, 23 and 29 (iii) 8, 9 and 25

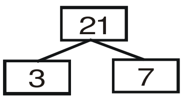
Sol. (i) 12, 15 and 21



So, $12 = 2 \times 2 \times 3 = 2^2 \times 3$



So, $15 = 3 \times 5$



So, $21 = 3 \times 7$

Therefore, HCF (12, 15, 21) = 3 ; LCM = (12, 15, 21) $= 2^2 \times 3 \times 5 \times 7 = 420$

(Rest Try your self)

Ex.9 Given that HCF (306, 657) = 9, find LCM (306, 657).

Sol. $$\text{LCM }(306, 657) = \frac{306 \times 657}{\text{HCF }(306, 657)} = \frac{306 \times 657}{9} = 22338.$$

Ex.10 Prove that square of any odd integer is of the form 8k + 1, k an integer.

Sol. For any odd integer is of the form 2m + 1 and $(2m + 1)^2 = 4m^2 + 4m + 1$

$= 4\,m\,(m + 1) + 1$

Now, m (m + 1) is alwyas even, say 2k, hence $4m(m + 1) = 4 \times 2k$

$\Rightarrow \quad (2m + 1)^2 = 8k + 1$

Ex.11 Check whether 6^n can end with the digit 0 for any natural number n.

Sol. If the number 6^n, for any natural number n, ends with digit 0, then it would be divisible by 5. That is, the prime factorisation of 6^n would contain the prime number 5. This is not possible because $6^n = (2 \times 3)^n = 2^n \times 3^n$; so the only primes in the factorisation of 6^n are 2 and 3 and the uniqueness of the Fundamental Theorem of Arithmetic guarantees that there are no other primes in the factorisation of 6^n. So, there is no natural number n for which 6^n ends with the digit zero.

Ex.12 Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Sol. (i) $7 \times 11 \times 13 + 13 = (7 \times 11 + 1) \times 13$

$= (77 + 1) \times 13$

$= 78 \times 13$

$= (2 \times 3 \times 13) \times 13$ $[\because 78 = 2 \times 3 \times 13]$

$= 2 \times 3 \times 13^2$

Since, $7 \times 11 \times 13 + 13$ can be expressed as a product of primes, therefore, it is a composite number.

(Rest Try your self)

Ex.13 Prove that $\sqrt{5}$ is irrational.

Sol. Let us assume, to the contrary, that $\sqrt{5}$ is rational.

So, we can find coprime integers a and b ($\neq 0$) such that

$$\sqrt{5} = \frac{a}{b}$$

$\Rightarrow$ $\sqrt{5}\,b = a$

Squaring on both sides, we get

$$5b^2 = a^2$$

Therefore, 5 divides a^2.

Therefore, 5, divides a

So, we can write $a = 5c$ for some integer c.

Substituting for a, we get

$$5b^2 = 25c^2$$

$\Rightarrow$ $b^2 = 5c^2$

This means that 5 divides b^2, and so 5 divides b.

Therefore, a and b have at least 5 as a common factor.

But this contradicts the fact that a and b have no common factor other than 1.

This contradiction arose because of our incorrect assumption that $\sqrt{5}$ is rational.

So, we conclude that $\sqrt{5}$ is irrational.

Ex.14 There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Sol. [Hint : Take LCM of 18 and 12]

Ex.15 Prove that $3 + 2\sqrt{5}$ is irrational.

Sol. Let us assume, to the contrary, that $3 + 2\sqrt{5}$ is rational.

That is, we can find coprime integers a and b ($b \neq 0$) such that $3 + 2\sqrt{5} = \frac{a}{b}$

Therefore, $\frac{a}{b} - 3 = 2\sqrt{5}$

$$\Rightarrow \quad \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \quad \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2} = \sqrt{5}$$

Since a and b are integers, we get $\frac{a}{2b} - \frac{3}{2}$ is rational, and so $\frac{a-3b}{2b} = \sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational. This contradiction has arisen because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

Ex.16 Show that $5 - \sqrt{3}$ is irrational.

Sol. Suppose $5 - \sqrt{3}$ is a rational number (say p)

Then $5 - \sqrt{3} = p$

$\Rightarrow 5 - p = \sqrt{3}$

Since p is a rational number and 5 is a rational

$\therefore$ 5 – p being the difference of two rational numbers is a rational number because the set of rational numbers are closed w.r.t. the operation of subtraction.

$\Rightarrow \sqrt{3}$ is a rational number But, it is an irrational number

$\Rightarrow$ Our supposition is wrong. Hence $5 - \sqrt{3}$ irrational .

Ex.17 Prove that the following are irrationals :

(i) $\frac{1}{\sqrt{2}}$ (ii) $7\sqrt{5}$ (iii) $6 + \sqrt{2}$

Sol. (Try yourself)

Ex.18 Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

(i) $\frac{13}{3125}$ (ii) $\frac{17}{8}$ (iii) $\frac{64}{455}$ (iv) $\frac{15}{1600}$ (v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$ (vii) $\frac{129}{2^2 5^7 7^5}$ (viii) $\frac{6}{15}$ (ix) $\frac{35}{50}$ (x) $\frac{77}{210}$

Sol. (i) $\frac{13}{3125} = \frac{13}{5^5}$

Hence, $q = 5^5$, which is of the form $2^n 5^m$ (n = 0, m = 5). So, the rational number $\frac{13}{3125}$ has a terminating decimal expansion.

(ii) $\frac{17}{8} = \frac{17}{2^3}$

Hence, $q = 2^3$, which is of the form $2^n 5^m$ (n = 3, m = 0). So, the rational number $\frac{17}{8}$ has a terminating decimal expansion.

(iii) $\frac{64}{455} = \frac{64}{5 \times 7 \times 13}$

Hence, $q = 5 \times 7 \times 13$, which is not of the form $2^n 5^m$. So, the rational number $\frac{64}{455}$ has a non-terminating repeating decimal expansion. **(Rest Try your self)**

Ex.19 Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Sol. (i) $\frac{13}{3125}$

$$= \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{416}{10^5} = 0.00416$$

(ii) $\frac{17}{8} = \frac{17}{2^3}$

$$= \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.125$$

(Rest Try your self)

Ex.20 The following real numbers have decimal expansions as given below. In each case, decide whether they are rational, or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

(i) 43.123456789

(ii) 0.120 1200 12000 120000....

(iii) $\mathbf{43.\overline{123456789}}$

Sol. (i) 43.123456789

Since, the decimal expansion terminates, so the given real number is rational and therefore of the form $\frac{p}{q}$. 43.123456789

$$= \frac{43123456789}{1000000000}$$

$$= \frac{43123456789}{10^9}$$

$$= \frac{43123456789}{(2 \times 5)^9}$$

$$= \frac{43123456789}{2^9\,5^9}$$

Hence, $q = 2^9\,5^9$

The prime factorization of q is of the form $2^n\,5^m$, where n = 9, m = 9.

(ii) 0.120 1200 12000 120000....

Since, the decimal expansion is neither terminating nor non-terminating repeating, therefore, the given real number is not rational.

(Rest Try your self)

EXERCISE – I UNSOLVED PROBLEMS

Q.1 Prove that $\sqrt{2}$ is not a rational number or there is no rational whose square is 2.

Q.2 Prove that $\sqrt[3]{3}$ is irrational.

Q.3 Prove that $2 + \sqrt{3}$ is irrational.

Q.4 Prove that $\sqrt{2}+\sqrt{3}$ is irrational.

Q.5 Use Euclid's division lemma to show that the square of any positive integer is either of the form 3m or 3m +1 for some interger m.

Q.6 Show that one and only one out of n, n + 2 or n + 4 is divisible by 3, where n is any positive integer.

Q.7 Use Euclid's algorithm to find the HCF of 4052 and 12576.

Q.8 Find the HCF of 1848, 3058 and 1331.

Q.9 Using Euclid's division, find the HCF of 56, 96 and 404

Q.10 Find the L.C.M and H.C.F. of 1296 and 2520 by applying the fundamental theorem of arithmetic method i.e. using the prime factorisation method.

Q.11 Given that H.C.F. (336, 54) = 6. Find L.C.M. (336, 54)

Q.12 Given that L.C.M. (150, 100) = 300, find H.C.F. (150, 100).

Q.13 State Euclid's division lemma.

Q.14 State fundamental theorem of Arithmetic.

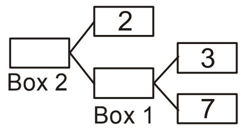
Q.15 For any two positive integers, state the relation between the numbers, their H.C.F. and L.C.M. Is this result true for three positive integers?

Q.16 Explain why $7 \times 11 \times 13 + 13$ is a composite number

Q.17 If $\frac{p}{q}$ is a rational number $(q \neq 0)$, what is the condition on q so that the decimal representation of $\frac{p}{q}$ is terminating?

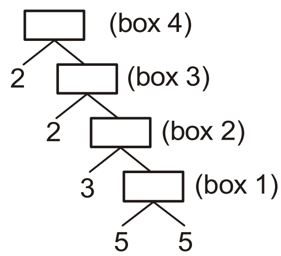
Q.18 Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Q.19 Complete the missing entries in the following factor tree?



Q.20 Find the prime factors of 560.

Q.21 Find the missing numbers in the following factorisation :



Q.22 Find the H.C.F. and L.C.M. of 17, 23, 37 by applying prime factorisation method.

Q.23 Given that H.C.F. (14, 35) = 7, find L.C.M. (14, 35).

Q.24 Without performing long division, state whether the number $\frac{84}{455}$ will have a terminating or a non-terminating but repeating decimal representation.

Q.25 Write down the decimal representation of $\frac{77}{210}$.

Q.26 The following real numbers have decimal expansion as given below. In each case decide whether it is rational or not. If rational, what can be said about the prime factors of denominator.

(i) 4.59

(ii) 0.13013001300013000013......

(iii) $0.0\overline{6012}$

Q.27 Find two irrational numbers between 0.1 and 0.12.

Q.28 Find a rational number and also an irrational number between the numbers.

a = 0.101001000100001......,

b = 0.1001000100001......

Q.29 Show that any positive odd integer is of the form 4q + 1 or 4q + 3, where $q \in Z^+$.

Q.30 Without actually performing the long division, state whether the rational number $\frac{17}{1600}$ will have terminating or non-terminating but repeated decimal representation.

Q.31 Examine whether the following numbers are rational or irrational :

(i) $\left(2-\sqrt{3}\right)^2$ (ii) $\left(\sqrt{2}+\sqrt{3}\right)^2$

(iii) $\left(3-\sqrt{3}\right)\left(3+\sqrt{3}\right)$ (iv) $\frac{2\sqrt{3}}{3}$

Q.32 Show that $p^2 - 1$ is divisible by 8, where p is an odd positive interger.

Q.33 Prove that $\sqrt{5}$ is an irrational number.

Q.34 Use Euclid's division lemma to show that the cube of any positive integer is of the form 9m, 9m + 1 or 9m + 8.

Q.35 Check whether 7^n can end with digit 0, for any natural number n.

Q.36 If H.C.F. (65, 117) is expressed in the form 65m + 117n, then find the value of m.

Find the g.c.d. of the following pairs of polynomials

Q.37 $4(x - 3)^2 (x - 1) (x + 1)^3$ and $6(x - 1)^2 (x + 1)^2 (x + 7)$

Q.38 $(x - 2)^2 (x + 3) (x - 4)$ and $(x - 2) (x + 2) (x - 5)$

Q.39 $(2x - 7) (3x + 4)$ and $(2x - 7)^2 (x + 3)$

Q.40 $(3x - 2)^2 (2x + 3)^3 (x - 1)$ and $(3x - 2)^3 (2x + 3) (x - 1)^3$

Q.41 $(x - 1) (x + 1)^3$ and $(x - 1)^3 (x + 1)$

Q.42 $(x + 4)^2 (x - 3)^3$ and $(x - 1) (x + 4) (x - 3)^2$

Q.43 $24(x - 3) (x - 2)^2$ and $15(x - 2) (x - 3)^3$

Q.44 $16 - 4x^2$ and $x^2 + x - 6$

Q.45 $xy - y$ and $x^4y - xy$

Q.46 $2x^2 - 7x + 3$ and $3x^2 - 7x - 6$

Q.47 $x^3 + 64$ and $x^2 - 16$

Q.48 $x^3 + 2x^2 - 3x$ and $2x^3 + 5x^2 - 3x$

Q.49 $22x(x + 1)^2$ and $36x^2(2x^2 + 3x + 1)$

Q.50 $3 + 13x - 30x^2$ and $25x^2 - 30x + 9$

Q.51 $(x^3 - y^3)$ and $(x^4 + x^2y^2 + y^4)$

Q.52 $2x^2 + 7xy + 3y^2$ and $2x^2 + 6xy + x + 3y$

Q.53 $56(x^6y^2 - x^2y^6)$ and $72(x^5y^3 + 3y^5x^3 + 2y^7x)$

Q.54 $54(x^3 + 8y^3)$ and $90(x^3 + 7x^2y + 16xy^2 + 12y^3)$

Q.55 $(4x^4 + y^4)$ and $(2x^3 - xy^2 - y^3)$

Q.56 $x^3 - y^3$; $x^3 y - y^4$ and $y^2(x - y)^2 (x^2 + xy + y^2)$

Q.57 $(2x^2 - 3xy)^2$; $(4x - 6y)^3$ and $(8x^3 - 27y^3)$

Q.58 $(4x^4 + y^4)$; $(2x^3 - xy^2 - y^3)$ and $(2x^2 + 2xy + y^2)$

Q.59 $x^4 - x^3 + x - 1$ and $x^4 + x^2 + 1$

Q.60 $(8x^6 - 32x^5 + 128x^3 - 128x^2)$ and $(12x^6 - 36x^5 + 48x^3)$

Q.61 $(x^3 - x^2 - x + 1)$ and $(x^4 - 2x^3 + 2x - 1)$

Q.62 $2x^2y(x^2 - y^2)$ and $35\ xy^2(x - y)$

Q.63 $(x^2 + 4x - 21)$ and $(x^3 + 7x^2 - 9x - 63)$

Q.64 $(6x^4 - 13x^3 + 6x^2)$ and $(8x^4 - 36x^3 + 54x^2 - 27x)$

Q.65 $(x^3 - x^2 - x - 2)$ and $(x^3 + 3x^2 - 6x - 8)$

Q.66 $(4x^5 + 16x^4 - 44x^2 - 24x)$ and $(2x^5 - 6x^3 + 2x)$

Q.67 $(3x^3 - 14x^2 + 9x + 10)$ and $(15x^3 - 34x^2 + 21x - 10)$

Q.68 $4x^2(x^2 - a^2)$ and $9x^2(x^3 - a^3)$

Q.69 $2(a^2 - b^2)$ and $3(a^3 - b^3)$

Q.70 $(x^2 + 3x - 4)$ and $(x^3 - 2x^2 - 2x + 3)$

Q.71 $12(x^4 - 25)$ and $8(x^4 + 4x^2 - 5)$

Q.72 Find the values of a and b so that the polynomial $x^3 + ax^2 + bx - 42$ is divisible by $x^2 - x - 6$.

Q.73 Find the values of a and b so that the polynomial $x^3 + ax^2 + bx - 6$ is completely divisible by $x^2 - 4x - 3$.

Q.74 Find the values of a and b so that the polynomial $f(x) = 3x^3 + ax^2 - 13x + b$ is divisible by $x^2 - 2x - 3$.

Q.75 Find the GCD of the polynomials

(i) $12(x^3 + x^2 + x + 1)$ and $18(x^4 - 1)$.

(ii) $12(3x^4 - 14x^3 - 5x^2)$ and $30(3x^5 + 4x^4 + x^3)$

Q.76 Find the GCD of the polynomials :

(i) $x^3 + 2x^2 - 3x$ and $2x^3 + 5x^2 - 3x$

(ii) $4(x^4 - 1)$ and $6(x^3 - x^2 - x + 1)$

Q.77 If $x^2 - x - 6$ is the GCD of the expressions $(x + 2)(2x^2 + ax + 3)$ and $(x - 3)(3x^2 + bx + 8)$. Find the values of a and b.

Q.78 Find the GCD of the polynomials :

$2(x^4 - y^4)$, $3(x^2 + 2x^2y - xy^2 - 2y^3)$.

Q.79 If $(x^2 - x - 2)$ is the GCD of the expressions $(x - 2)(2x^2 + ax + 1)$ and $(x + 1)(3x^2 + bx + 2)$, find the values of a and b.

Q.80 Find the GCD of the polynomials :

(i) $p(x) = 45(2x^4 - x^3 - x^2)$ and $q(x) = 75(8x^5 + x^2)$.

(ii) $p(x) = 36(3x^4 + 5x^3 - 2x^2)$ and $q(x) = 54(27x^4 - x)$.

(iii) $p(x) = 42(2x^3 - 5x^2 - 3x)$ and $q(x) = 60(8x^4 + x)$.

Q.81 $(x + 1)(x - 4)$ is the g.c.d. of the polynomials $(x - 4)(2x^2 + x - a)$ and $(x + 1)(2x^2 + bx - 12)$. Find a and b.

Q.82 $(x - 3)$ is the g.c.d. of $x^3 - 2x^2 + px + 1$ and $x^2 - 5x + q$. Find the $6p + 5q$.

Q.83 For what value of k, the g.c.d. of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$ is $x + 4$?

Q.84 If G.C.D. of $(x - 5)(x^2 - x - a)$ and $(x - 4)(x^2 - 2x - b)$ is $(x - 4)(x - 5)$, find the values of a, b.

Q.85 Find the values of k for which the g.c.d. of $x^2 - 2x - 24$ and $x^2 - kx - 6$ is $x - 6$.

Q.86 Find the H.C.F. of $x^4 - 1$ and $x^3 + x^2 + x + 1$.

Q.87 Find the value of a and b so that the polynomials P(x) and Q(x) have $(x + 1)(x - 2)$ as their HCF :

$P(x) = (x^2 + 3x + 2)(x^2 + x + a)$

$Q(x) = (x^2 - 3x + 2)(x^2 - 3x + b)$

Q.88 Find the values of a and b such that the polynomials. P(x) and Q(x) have $(x + 3)(x - 2)$ as their HCF :

$P(x) = (x^2 - 4x - 21)(x^2 - 4x + a)$

$Q(x) = (x^2 - 5x + 6)(x^2 - 4x + b)$

Q.89 If $(x - \alpha)$ is the GCD of $ax^2 + bx + c$ and $bx^2 + cx + a$, prove that either

$a = 0$ or $a^3 + b^3 + c^3 = 3abc$.

Q.90 If $(x - \alpha)$ is the GCD of $ax^2 + bx + c$ and $cx^2 + bx + a$, prove that

$a + b + c = 0$ or $a - b + c = 0$

Q.91 If the polynomials $x^2 - 4mx + 2$ and $3x^2 - 2mx - 4$ have a common linear factor, find the values of m.

Q.92 If $(x - \alpha)$ is the HCF of $px^2 + qx + c$ and $ax^2 + bx + c$, prove that $\alpha(p - a) = (b - q)$.

Q.93 Find the HCF (GCD) of the polynomials :

(i) $(x - 3)(x + 5)^2$, $(x + 5)(x + 7)^2$ and $(x + 2)(x + 5)^3$

(ii) $2(x - 7)(x + 7)^2$, $4(x - 7)^2(x + 8)$ and $8(x^2 - 49)$

(iii) $x^2 - 1$, $x^4 - 1$ and $(x - 1)^2$

(iv) $4(x + 2)$, $8(x + 3)$ and $16(x + 4)$

(v) $3(x + 3)$, $5(x + 3)(x + 5)$ and $7(x + 4)$

Q.94 Fine what value(s) of k is the HCF of $x^2 + x - (2k + 2)$ and $2x^2 + kx - 12$, $(x + 4)$?

Q.95 If the HCF of the polynomials

$f(x) = (x - 1)(x^2 + 3x + a)$ and,

$g(x) = (x + 2)(x^2 + 2x + b)$ is $(x - 1)(x + 2)$

find a and b.

Q.96 Find a and b so that the polynomials

$f(x) = (x^2 + 3x + 2)(x^2 + 2x + a)$

and $g(x) = (x^2 + 7x + 12)(x^2 + 7x + b)$ may have $(x + 1)(x + 3)$ as their HCF.

Q.97 If $(x - 2)$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b.

Q.98 If $(x + 3)(x - 2)$ is the GCD of $f(x) = (x + 3)(2x^2 - 3x + a)$ and, $g(x) = (x - 2)(3x^2 + 10x - b)$ find the values of a and b.

ANSWER KEY

7. HCF = 4 **8.** HCF = 11 **9.** HCF = 4

10. HCF = 72, LCM = 45360 **11.** LCM = 3024

12. HCF = 50 **15.** HCF × LCM = a × b, No

16. More than 2 factors

17. Factors of q are in the form of 2^m or 5^n or $2^m \times 5^n$

18. 1.5 **19.** 21, 42

20. $2^4 \times 5 \times 7$ **21.** 25, 75, 150, 300

22. 1,14467 **23.** 70 **24.** Non-terminating

25. $0.3\overline{6}$

26. (i) Rational **(ii)** Not Rational **(iii)** Rational

27. 0.101001000...., 0.1121231234....

28. 0.1002, 0.100212112111....

30. Terminating

31. (i) Irrational **(ii)** Irrational

(iii) Rational **(iv)** Irrational

35. Not **36.** m = 2, n = –1

37. $2(x - 1)(x + 1)^2$ **38.** $(x - 2)$

39. $(2x - 7)$ **40.** $(3x - 2)^2(2x + 3)(x - 1)$

41. $(x - 1)(x + 1)$ **42.** $(x + 4)(x - 3)^2$

43. $3(x - 3)(x - 2)$ **44.** $(x - 2)$

45. $(x - 1)y$ **46.** $(x - 3)$ **47.** $(x + 4)$

48. $x(x + 3)$ **49.** $2x(x + 1)$ **50.** $(5x - 3)$

51. $x^2 + xy + y^2$ **52.** $x + 3y$

53. $8xy^2(x^2 + y^2)$ **54.** $18(x + 2y)$

55. $2x^2 + 2xy + y^2$ **56.** $x^3 - y^3$ **57.** $2x - 3y$

58. $2x^2 + 2xy + y^2$ **59.** $x^2 - x + 1$ **60.** $4x^2(x - 2)$

61. $(x - 1)^2(x + 1)$ **62.** $xy(x - y)$

63. $(x + 7)(x - 3)^2$ **64.** $x(2x - 3)$ **65.** $(x - 2)$

66. $2x(x^2 - x - 1)$ **67.** $(3x - 5)$ **68.** $x^2(x - a)$

69. $(a - b)$ **70.** $(x - 1)$ **71.** $4(x^2 + 5)$

72. a = 6, b = – 13 **73.** a = – 6, b = 11

74. a = – 4, b = – 6

75. (i) $6(x + 1)(x^2 + 1)$ (ii) $6x^2(3x + 1)$

76. (i) $x(x + 3)$ (ii) $2(x^2 - 1)$

77. a = – 7, b = 10 **78.** $x^2 - y^2$ **79.** a = 3, b = – 7

80. (i) $15x^2(2x + 1)$ (ii) $18x(3x - 1)$ (iii) $6x(2x + 1)$

81. a = 1, b = – 5 **82.** 10 **83.** k = 5

84. a = 12, b = 15 **85.** k = 5 **86.** $(x + 1)(x^2 + 1)$

87. a = – 6, b = – 4 **88.** a = 4, b = – 21

91. $m = \pm\frac{1}{\sqrt{2}}$

93. (i) $x + 5$ (ii) $2(x - 7)$ (iii) $x - 1$ (iv) 4 (v) 1

94. 5 **95.** a = 2, b = – 3

96. a = – 3, b = 6 **97.** a = – 3, b = 6

98. a = – 2, b = 3.

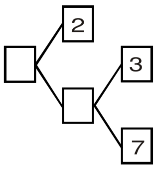
EXERCISE – II | BOARD PROBLEMS

Questions Carrying 1 Mark

Q.1 If $\frac{p}{q}$ is a rational number ($q \neq 0$), what is condition of q so that the decimal representation of $\frac{p}{q}$ is terminating?

Q.2 Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Q.3 Complete the missing entries in the following factor tree :



Q.4 The decimal expansion of the rational number $\frac{43}{2^4 \cdot 5^3}$, will terminate after how many places of decimals?

Q.5 Find the [HCF × LCM] for the numbers 100 and 190.

Q.6 Find the [HCF × LCM] for the numbers 105 and 120.

Q.7 Write whether the rational number $\frac{51}{1500}$ will have a terminating decimal expansion or a non-terminating repeating decimal expansion.

Q.8 The HCF and LCM of two numbers are 9 and 360 respectively. If one number is 45, write the other number.

Questions Carrying 3 Marks

Q.9 Show that $5 - 2\sqrt{3}$ is an irrational number.

Q.10 Show that $2 - \sqrt{3}$ is an irrational number.

Q.11 Show that $5 + 3\sqrt{2}$ is an irrational number.

Q.12 Prove that $\sqrt{3}$ is an irrational number.

Q.13 Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or 3m + 1 for some integer m.

Q.14 Prove that $\sqrt{2}$ is an irrational number.

Q.15 Prove that $\sqrt{5}$ is an irrational number.

Q.16 Prove that $3 + \sqrt{2}$ is an irrational number.

Q.17 Prove that $3 + 5\sqrt{2}$ is an irrational number.

Q.18 Show that the square of any positive odd integers is of the form 8m + 1, for some integer m.

Q.19 Prove that $7 + 3\sqrt{2}$ is not a rational number.

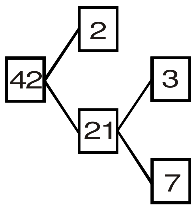
ANSWER KEY

1. $q = 2^n \times 5^m$, where n and m are whole numbers.

2. $\sqrt{2} = 1.41......., \sqrt{3} = 1.73.....$

$\therefore$ One rational no. between $\sqrt{2}$ and $\sqrt{3}$ is 1.5.

3.



4. After 4 decimal; $\frac{43}{2^4 \cdot 5^3} = \frac{43}{2000} = 0.0215$

5. HCF × LCM = 100 × 190 = 19000

6. HCF × LCM = 105 × 120 = 12600

7. $\frac{51}{1500} = \frac{17}{500}$; $500 = 2^2 \times 5^3$ ($2^m \cdot 5^n$). So, it has terminating expansion.

8. Other number = $\frac{9 \times 360}{45} = 72$

EXERCISE – III NTSE /OLYMPIAD /FOUNDATION PROBLEMS

Q.1 A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is

(A) $\frac{7}{24}$ (B) 0.29

(C) $\frac{13}{48}$ (D) all the above

Q.2 An irrational number is

(A) a terminating and nonrepeating decimal

(B) a nonterminating and non repeating decimal

(C) a terminating and repeating decimal

(D) a nonterminating and repeating decimal

Q.3 Which of the following statements is true

(A) Every point on the number line represents a rational number

(B) Irrational numbers cannot be represented by points on the number line

(C) $\frac{22}{7}$ is a rational number

(D) None of these

Q.4 The set of real numbers does not satisfy the property of

(A) multiplicative inverse

(B) additive inverse

(C) multiplicative identity

(D) none of these

Q.5 If m is an integer, then square of any positive integer is of the form :

(A) 2m + 1 (B) 2 m or 3m

(C) 3m or 3m + 1 (D) 2m + 1 or 3m + 1

Q.6 If 'm' is an irrational number then '2m' is

(A) a rational number

(B) an irrational number

(C) a whole number

(D) a natural number

Q.7 The value of $\sqrt{3}$ is

(A) 0.414 (B) 2.256

(C) 1.732 (D) none

Q.8 The sum of a rational and an irrational number.

(A) an irrational number

(B) a rational number

(C) an integer

(D) a whole number

Q.9 The product of two irrationals is

(A) a rational number

(B) an irrational number

(C) either A or B

(D) neither A nor B

Q.10 The value of $1.\overline{34} + 4.1\overline{2}$ is

(A) $\frac{133}{99}$ (B) $\frac{371}{90}$

(C) $\frac{5169}{990}$ (D) $\frac{5411}{990}$

Q.11 Which of the following statements is false?

(A) Every fraction is a rational number

(B) Every rational number is a fraction

(C) Every integer is a rational number

(D) All the above

Q.12 An irrational number is :

(A) a terminating and non-repeating decimal

(B) a non-terminating and non-repeating decimal

(C) a terminating and repeating decimal

(D) a non-terminating and repeating decimal

Q.13 7 × 11 × 13 + 13 and 7 × 6 × 5 × 4 × 3 × 2 × 1 + 5 are

(A) composite numbers

(B) whole number

(C) prime numbers

(D) none of these

Q.14 HCF of two numbers is 113, their LCM is 56952. If one number is 904, the other number is :

(A) 7719 (B) 7119

(C) 7791 (D) 7911

Q.15 For what least value of n (where n is a natural number), $(24)^n$ is divisible by 8?

(A) 0

(B) −1

(C) 1

(D) no value of x possible

Q.16 π is

(A) a rational

(B) an irrational

(C) both (A) and (B)

(D) sometimes rational, sometimes irrational

Q.17 Expressing $0.3\overline{58}$ as a rational number, we get :

(A) $\frac{358}{100}$ (B) $\frac{358}{999}$

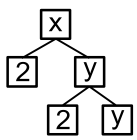
(C) $\frac{355}{990}$ (D) None of these

Q.18 Which of the following numbers has the terminating decimal representation ?

(A) $\frac{1}{7}$ (B) $\frac{1}{3}$

(C) $\frac{3}{5}$ (D) $\frac{17}{6}$

Q.19 Missing numbers in the following factor tree are :



(A) x = 34, y = 68 (B) y = 34, x = 68

(C) y = 34, x = 34 (D) y = 68, x = 68

Q.20 A lemma is an axiom used for proving :

(A) other statement

(B) no statement

(C) contradictory statement

(D) none of these

Q.21 If $a = \frac{1}{3-2\sqrt{2}}$, $b = \frac{1}{3+2\sqrt{2}}$, then the value of $a^2 + b^2$ is :

(A) 34 (B) 35

(C) 36 (D) 37

Q.22 The set of all irrational number is closed for

(A) addition (B) multiplication

(C) division (D) none of these

Q.23 The additive inverse of $\frac{-a}{b}$ is

(A) $\frac{a}{b}$ (B) $\frac{b}{a}$

(C) $\frac{-b}{a}$ (D) $\frac{-a}{b}$

Q.24 Multiplicative inverse of '0' is

(A) $\frac{1}{0}$ (B) 0

(C) does not exist (D) none of these

Q.25 If $\sqrt{5} = 2.236$ and $\sqrt{3} = 1.732$, then the value of $\frac{2}{\sqrt{5}+\sqrt{3}} + \frac{7}{\sqrt{5}-\sqrt{3}}$ is

(A) 14 (B) 14.39

(C) 14.392 (D) 16

Q.26 A rational number can be expressed as a terminating decimal if the denominator has factors

(A) 2 or 5 (B) 2, 3 or 5

(C) 3 or 5 (D) none of these

Q.27 Express 0.75 as rational number.

(A) $\frac{75}{99}$ (B) $\frac{75}{90}$

(C) $\frac{3}{4}$ (D) None

Q.28 Express $0.\overline{75}$ as rational number.

(A) $\frac{75}{90}$ (B) $\frac{25}{33}$

(C) $\frac{3}{4}$ (D) None

Q.29 Express $0.3\overline{58}$ as rational number

(A) $\frac{358}{1000}$ (B) $\frac{358}{999}$

(C) $\frac{355}{990}$ (D) All

Q.30 Which of the following statements is true?

(A) $\frac{5}{7}<\frac{7}{9}<\frac{9}{11}<\frac{11}{13}$

(B) $\frac{11}{13}<\frac{9}{11}<\frac{7}{9}<\frac{5}{7}$

(C) $\frac{5}{7}<\frac{11}{13}<\frac{7}{9}<\frac{9}{11}$

(D) $\frac{5}{7}<\frac{9}{11}<\frac{11}{13}<\frac{7}{9}$

Q.31 _______ is a series of well defined steps, which gives a procedure for solving a type of problem.

(A) Theorem (B) Statment

(C) Algorithm (D) None of these

Q.32 Euclid's division algorithm can be applied to :

(A) only positive integers

(B) only negative integers

(C) all integers

(D) none of these

Q.33 Expressing 0.625 as a rational number, we get :

(A) $\frac{62.5}{1000}$ (B) $\frac{5}{8}$

(C) $\frac{625}{100}$ (D) None of these

Q.34 A rational number in its standard from can be expressed as a terminating decimal, if the denominator has factors :

(A) 2 or 5 (B) 2, 3 or 5

(C) 3 or 5 (D) None of these

Q.35 $0.3\overline{6}$ as a fraction in the simplest form is :

(A) $\frac{36}{90}$ (B) $\frac{36}{100}$

(C) $\frac{11}{30}$ (D) $\frac{33}{90}$

Q.36 $0.2\overline{54}$ as a fraction in the simplest form is

(A) $\frac{14}{55}$ (B) $\frac{254}{1000}$

(C) $\frac{42}{165}$ (D) $\frac{126}{495}$

Q.37 Representation of $3.\overline{6}$ as a fraction in the simplest form is :

(A) $\frac{11}{13}$ (B) $\frac{11}{3}$

(C) $\frac{36}{10}$ (D) $\frac{3}{11}$

Q.38 $5+\sqrt{6}$ is :

(A) a rational number

(B) an irrational number

(C) can't say

(D) both (A) and (B)

Q.39 $0.\overline{16}$ as rational number in the simplest form is :

(A) $\frac{16}{1000}$ (B) $\frac{16}{99}$

(C) $\frac{16}{100}$ (D) $\frac{8}{50}$

Q.40 0.1010010001 ...... is :

(A) a rational number

(B) a repeating decimal number

(C) an irrational number

(D) both (A) and (C)

Q.41 The value of $4-\cfrac{5}{1+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{4}}}}$ is

(A) $\frac{40}{31}$ (B) $\frac{4}{9}$

(C) $\frac{1}{8}$ (D) $\frac{31}{40}$

Q.42 The sum of the additive inverse and multiplicative inverse of 2 is

(A) $\frac{3}{2}$ (B) $\frac{-3}{2}$

(C) $\frac{1}{2}$ (D) $\frac{-1}{2}$

Q.43 If $\sqrt{6}$ = 2.449 then the value of $\frac{3\sqrt{2}}{2\sqrt{3}}$ is close to

(A) 1.225 (B) 0.816

(C) 0.613 (D) 2.449

Q.44 The value of $\sqrt{5\sqrt{5\sqrt{5\sqrt{5......}}}}$ is

(A) 0
(B) 5
(C) can't be determined
(D) none

Q.45 The greatest among the following is

I. $\sqrt[3]{1.728}$ II. $\frac{\sqrt{3}-1}{\sqrt{3}+1}$

III. $\left(\frac{1}{2}\right)^{-2}$ IV. $\frac{17}{8}$

(A) I (B) IV
(C) II (D) III

Q.46 Which of the following is not a rational number?

(A) $\sqrt{2}$ (B) $\sqrt{4}$
(C) $\sqrt{9}$ (D) $\sqrt{16}$

Q.47 Set of natural numbers is a subset of
(A) set of even numbers
(B) set of odd numbers
(C) set of composite numbers
(D) set of real numbers

Q.48 The quotient of $(3+2\sqrt{3})\div(3-2\sqrt{3})$ is :
(A) a rational
(B) an irrational
(C) both (A) and (B)
(D) either (A) or (B)

Q.49 Which of the following is a rational number

(A) $\sqrt{5}$ (B) $\sqrt{6}$
(C) $\sqrt{8}$ (D) $\sqrt{9}$

Q.50 The sum of two rational number is always
(A) an irrational number
(B) a rational number
(C) a fraction
(D) none of these

Q.51 The product of a rational and an irrational number is always :
(A) a irrational number
(B) a rational number
(C) a fraction
(D) none of these

Q.52 The sum of a rational number and an irrational number is always :
(A) an irrational number
(B) a rational number
(C) an integer
(D) a whole number

Q.53 The product of two irrational number is :
(A) a rational number
(B) an irrational number
(C) either (A) or (B)
(D) neither (A) nor (B)

Q.54 $\frac{1}{\sqrt{2}}$ is :

(A) a rational
(B) an irrational
(C) both (A) and (B)
(D) none of these

Q.55 If q is some integer, then any positive odd integer is of the form :
(A) q – 1
(B) 2q
(C) 4q + 1 or 4q – 1
(D) 4q + 1 or 4q + 3

Q.56 If p : All integers are rational number and q : Every rational is an integer, then which of the following statements is correct?
(A) p is false and q is true
(B) p is true and q is false
(C) both p and q are true
(D) both p and q are false

Q.57 For any two rational number x and y, which of the following properties are correct?

(i) $x < y$ (ii) $x = y$ (iii) $x > y$

(A) only (i) and (ii) are correct
(B) only (ii) and (iii) are correct
(C) only (ii) is correct
(D) All (i), (ii), (iii) are correct

Q.58 If A : Every whole number is a natural number and R : 0 is not a natural number, then which of the following statement is true?

(A) A is false and R is the correct explanation of A

(B) A is true and R is the correct explanation of A

(C) A is true and R is false

(D) both A and R are true

Q.59 If x = $(3+\sqrt{8})$, then the value of $\left(x^2+\frac{1}{x^2}\right)$ is :

(A) 24 (B) 25

(C) 30 (D) 34

Q.60 The numbre $(6+\sqrt{2})(6-\sqrt{2})$ is

(A) rational (B) irrational

(C) can't say (D) none

Q.61 In a morning walk three persons step off together. Their steps measures 80 cm, 85 cm and 90 cm respectively. The minimum distance each should walk so that they can cover the distance in complete step is.

(A) 122 m 40 cm (B) 132 m 60 cm

(C) 125 m 31 cm (D) 120 m 40 cm

Q.62 The number $(\sqrt{2}+\sqrt{3})^2$ is

(A) rational number

(B) irrational number

(C) can't say

(D) none

Q.63 The number $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$,

where x, y > 0 is

(A) rational (B) irational

(C) both (D) none

Q.64 Two irrational numbers between $\sqrt{2}$ and $\sqrt{3}$ are :

(A) $2^{\frac{1}{2}}, 6^{\frac{1}{4}}$ (B) $3^{\frac{1}{4}}, 3^{\frac{1}{6}}$

(C) $6^{\frac{1}{8}}, 3^{\frac{1}{4}}$ (D) none of these

Q.65 Two tankers contain 850 litres and 680 litres of petrol respectively. The maximum capacity of a container which can measure the petrol of either tanker in exact number of times is :

(A) 160 litres (B) 168 litres

(C) 170 litres (D) 180 litres

Q.66 Rational number between $\sqrt{2}$ and $\sqrt{3}$ is

(A) $\frac{\sqrt{2}+\sqrt{3}}{2}$ (B) $\frac{\sqrt{2}\times\sqrt{3}}{2}$

(C) 1.5 (D) 1.8

Q.67 The irrational number between 2 and 3 is

(A) $\sqrt{2}$ (B) $\sqrt{3}$

(C) $\sqrt{5}$ (D) $\sqrt{11}$

Q.68 The number $\frac{3-\sqrt{3}}{3+\sqrt{3}}$ is

(A) rational (B) irrational

(C) both (D) can't say

Q.69 The rational number between $\frac{1}{2}$ and $\frac{1}{3}$ is

(A) $\frac{2}{5}$ (B) $\frac{1}{5}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Q.70 $\left(\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}\right)$ is equal to :

(A) 15 (B) 16

(C) 10 (D) 24

Q.71 $\sqrt[4]{\frac{1008}{63}}$ is equal to :

(A) 1 (B) 2

(C) 4 (D) 6

Q.72 $\sqrt[7]{5}$ is :

(A) an irrational

(B) a rational

(C) neither (A) nor (B)

(D) all of the above

Q.73 $(6+\sqrt{2})$ is :

(A) a rational (B) an irrational

(C) an integer (D) not real

Q.74 The greatest number of 6 digits exactly divisible by 24, 15 and 36 is :

(A) 999998 (B) 999999

(C) 999720 (D) 999724

Q.75 $\frac{121}{2^3 \times 3^2 \times 7^5}$ is

(A) a terminating decimal number

(B) a non-terminating repeating decimal

(C) a rational number

(D) both (B) and (C)

Q.76 The largest number of four digits exactly divisible by 12, 15, 18 and 27 is :

(A) 9720 (B) 9820

(C) 9920 (D) 9930

Q.77 $\frac{1}{3-\sqrt{8}}-\frac{1}{\sqrt{8}-\sqrt{7}}+\frac{1}{\sqrt{7}-\sqrt{6}}-\frac{1}{\sqrt{6}-\sqrt{5}}+\frac{1}{\sqrt{5}-2}$ is

(A) 0 (B) 1

(C) 2 (D) 5

Q.78 If a = 2 and b = 3, then $\left(\frac{1}{a}+\frac{1}{b}\right)^a$ is equal to :

(A) $\frac{4}{9}$ (B) $\frac{25}{36}$

(C) 17 (D) $\frac{31}{17}$

Q.79 The domain of the function

$f(x) = \sqrt{x-4}+\sqrt{x-5} + |x| + x^2$

(A) R – {4} (B) R – {4, 5}

(C) [5, ∞) (D) R

Q.80 The equivalent rational form of $17.\bar{6}$ is

(A) $\frac{53}{3}$ (B) $\frac{88}{5}$

(C) $\frac{44}{25}$ (D) none

Q.81 If p : Every fraction is a rational number and q : Every rational number is a fraction, then which of the following is correct?

(A) p is true and q is false

(B) p is false and q is true

(C) Both p and q are true

(D) Both p and q are false

Q.82 Which of the following is a rational number(s) ?

(A) $\frac{-2}{9}$ (B) $\frac{4}{-7}$

(C) $\frac{-3}{-17}$ (D) All the three

Q.83 $\frac{-3}{0}$ is a _____ .

(A) positive rational number

(B) negative rational number

(C) either positive or negative rational number

(D) neither positive nor negative rational number

Q.84 If x, y, z be rational number such that x > y and z < y then ....

(A) z > x (B) z < x

(C) y < z (D) y > x

Q.85 If A : The quotient of two integers is always a rational number and R : $\frac{1}{0}$ is not rational, then which of the following statement is true

(A) A is true and R is the correct explanation of A

(B) A is false and R is the correct explanation of A

(C) A is true and R is false

(D) Both A and R are false

Q.86 For what least value of n, where n is a natural number, 5^n is divisible by 3?

(A) 1 (B) 0

(C) 2 (D) no value of n is possible

Q.87 Which one is the correct alternative of the decimal representation of an irrational number ?

(A) non - terminating, non-repeating

(B) terminating

(C) termiating, repeating

(D) non-terminating, repeating

Q.88 The value of $1.\overline{34}+4.1\overline{2}$ is

(A) $\frac{133}{990}$ (B) $\frac{371}{290}$

(C) $\frac{5169}{990}$ (D) $\frac{5411}{990}$

Q.89 Which of the following numbers is the fourth power of a natural number?

(A) 6765207 (B) 6765201

(C) 6765206 (D) 6765209

Q.90 Two candles are of different lengths and thicknesses. The short and the long ones can burns respectively for 3.5 h and 5 h. After burning for 2 h, the lengths of the candles become equal in length. What fraction of the long candle's height was the short candle initially?

(A) $\frac{2}{7}$ (B) $\frac{5}{7}$

(C) $\frac{3}{5}$ (D) $\frac{4}{5}$

Q.91 I left home for bringing milk between 7 am and 8 am. The angle between the hour hand and the minute hand was 90°. I returned home between 7 am and 8 am. Then, also the angle between the minute hand and hour hand was 90°. At what time (nearest to second) did I leave and return home?

(A) 7 h 18 min 35 s and 7 h 51 min 24 s

(B) 7 h 19 min 24 s and 7 h 52 min 14 s

(C) 7 h 20 min 42 s and 7 h 53 min 11 s

(D) 7 h 21 min 49 s and 7 h 54 min 33 s

Q.92 The square of an odd integer must be of the form

(A) 6n + 1

(B) 6n + 3

(C) 8n + 3

(D) 4n + 1 but may not be 8n + 1

Q.93 $\sqrt{(a-b)^2}+\sqrt{(b-a)^2}$ is

(A) always zero

(B) never zero

(C) positive if and only, if a > b

(D) positivel only, if a ≠ b

Q.94 Which of the following is an irrational number?

(A) $\sqrt{41616}$

(B) 23.232323

(C) $\frac{\left(1+\sqrt{3}\right)^3-\left(1-\sqrt{3}\right)^3}{\sqrt{3}}$

(D) 23.10100100010000.....

ANSWER KEY

1.	D	**2.**	B	**3.**	D	**4.**	D
5.	C	**6.**	B	**7.**	C	**8.**	A
9.	C	**10.**	D	**11.**	B	**12.**	B
13.	A	**14.**	B	**15.**	C	**16.**	B
17.	C	**18.**	C	**19.**	B	**20.**	A
21.	A	**22.**	D	**23.**	A	**24.**	C
25.	C	**26.**	A	**27.**	C	**28.**	B
29.	C	**30.**	A	**31.**	C	**32.**	C
33.	B	**34.**	A	**35.**	C	**36.**	A
37.	B	**38.**	B	**39.**	B	**40.**	C
41.	C	**42.**	B	**43.**	A	**44.**	B
45.	D	**46.**	A	**47.**	D	**48.**	B
49.	D	**50.**	B	**51.**	A	**52.**	A
53.	C	**54.**	B	**55.**	D	**56.**	B
57.	D	**58.**	A	**59.**	D	**60.**	A
61.	A	**62.**	B	**63.**	A	**64.**	C
65.	C	**66.**	C	**67.**	C	**68.**	B
69.	C	**70.**	C	**71.**	B	**72.**	A
73.	B	**74.**	C	**75.**	D	**76.**	A
77.	D	**78.**	B	**79.**	C	**80.**	A
81.	A	**82.**	D	**83.**	D	**84.**	B
85.	B	**86.**	D	**87.**	A	**88.**	B
89.	B	**90.**	B	**91.**	D	**92.**	C
93.	D	**94.**	D				

POLYNOMIALS

BASIC CONCEPTS AND IMPORTANT RESULTS

Polynomials : The expression of the form $a_0x^n + a_1x^{n-1} + ...+ a_{n-1}x + a_n$ $a_0 \neq 0$ is called a polynomial. The expression $a_0x^n + a_1x^{n-1} + ...+ a_{n-1}x$ and a_n along with their sign are called the terms of the polynomial. The number $a_0, a_1, ..., a_n$ are real numbers and are called the coefficients of the terms of the polynomial.

Degree of polynomial : The highest power of x in p(x) is called the degree of polynomial.

(i) A polynomial of degree 0 is called a constant polynomial.

(ii) A polynomial of degree 1 is called a linear polynomial, for example, ax + b.

(iii) A polynomial of degree 2 is called a quadratic polynomial, for example, $ax^2 + bx + c$, $a \neq 0$

(iv) A polynomial of degree 3 is called cubic polynomial, for example, $ax^3 + bx^2 + cx + d$, $a \neq 0$

Monomials : Polynomials containing one term are called monomials.

Binomials : Polynomials containing two terms are called binomials

Trinomials : Polynomials containing three terms are called trinomials.

Zero of a polynomial : For any polynomial p(x), if p(k) = 0, k is called zero of polynomial p(x).

Number of zeros/roots of a polynomial p(x) is always equal or less than the highest degree of polynomial p(x).

Geometrical representation of monomials :

(i) Linear polynomial (ax + b) shows straight line.

(ii) Quadratic polynomial (ax^2 + bx + c) shows parabola :

(a) open upwards like if $a > 0$

(b) open downwards like if $a < 0$.

(iii) Polynomial of degree n crosses the x-axis at most n-points.

Remainder theorem : If p(x) be a polynomial is divided by (x – a), then remainder will be p(a) = r.

Factor theorem : If p(x) is divided by (x – a), then p(a) = 0.

Discriminant of a quadratic polynomial : If $p(x) = ax^2 + bx + c$ where $a \neq 0$, then $D = b^2 - 4ac$ is called discriminant of p(x).

(i) If $D > 0$, the graph of polynomial intersects x - axis at two distinct points. Points of intersection of x - axis are called zeroes of p(x).

(ii) If $D = 0$, the graph of polynomial touches x -axis at only one point. So there is only one zero of p(x).

(iii) If $D < 0$, the graph of polynomial neither touches nor intersects the x-axis. This shows p(x) does not have any zero.

Relationship between zeroes and coefficients of a quadratic polynomials :

(i) If α(alpha) and β(beta) are the zeroes of the polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then:

$$\alpha + \beta = -\frac{b}{a} = -\left(\frac{\text{coefficient of } x}{\text{coefficient of } x^2}\right) \text{ and}$$

$$\alpha\beta = \frac{c}{a} = \left(\frac{\text{constant term}}{\text{coefficient of } x^2}\right)$$

(ii) If α, β and γ are the zeroes of the polynomial $p(x) = ax^3 + bx^3 + cx + d$, then :

$$\alpha + \beta + \gamma = -\frac{b}{a} = -\left(\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3}\right)$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} = \left(\frac{\text{coefficient of } x}{\text{coefficient of } x^3}\right)$$

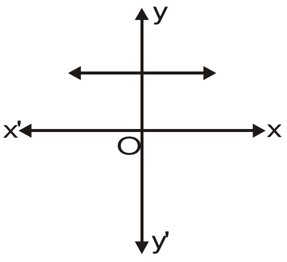
$$\alpha\beta\lambda = -\frac{d}{a} = -\left(\frac{\text{constant term}}{\text{coefficient of } x^3}\right)$$

Equation of quadratic polynomial : If α and β are the roots of a quadratic polynomial p(x), then p(x) quadratic equation be : $x^2 - (\alpha + \beta)x + \alpha\beta$.

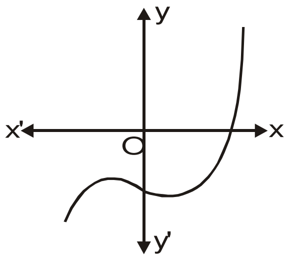
Division algorithm: If a polynomial p(x) is divided by another non-zero polynomial q(x), then there exist two polynomials g(x) and r(x) such that $p(x) = q(x) \times g(x) + r(x)$, where $r(x) = 0$ or degree of r(x) < degree of g(x). This is known as division algorithm

SOLVED PROBLEMS

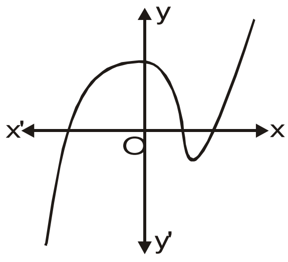
Ex.1 The graph of y = p(x) are given in fig below, for some polynomials p(x). Find the number of zeros of p(x), in each case.



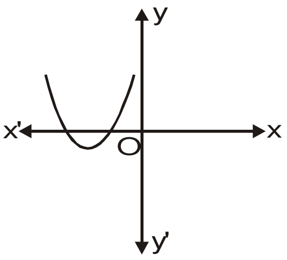
(i)



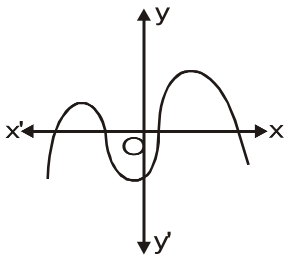
(ii)



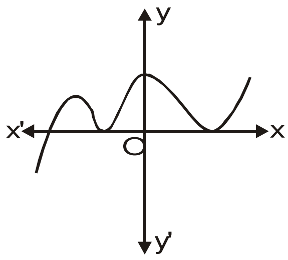
(iii)



(iv)



(v)



(vi)

Sol. (i) Graph of y = p(x) does not intersect the x-axis. Hence, polynomial p(x) has no zero.

(ii) Graph of y = p(x) intersects the x-axis at one and only one point.

Hence, polynomial p(x) has **one and only one** real zero. **(Rest Try your self)**

Ex.2 Find the zeros of the following quadratic polynomials and verify the relationship between the zeros and the coefficients.

(i) $x^2 - 2x - 8$ (ii) $4s^2 - 4s + 1$ (iii) $6x^2 - 3 - 7x$

(iv) $4u^2 + 8u$ (v) $t^2 - 15$ (vi) $3x^2 - x - 4$

Sol. (i) $x^2 - 2x - 8 = x^2 - 4x + 2x - 8 = x(x - 4) + 2(x - 4) = (x + 2)(x - 4)$

Zeroes are – 2 and 4.

Sum of the zeros $= (-2) + (4) = 2 = \dfrac{-(-2)}{1} = \dfrac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

Product of the zeros $= (-2)(4) = -8 = \dfrac{(-8)}{1} = \dfrac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$

(ii) $4s^2 - 4s + 1 = (2s - 1)^2$

The two zeros are $\dfrac{1}{2}, \dfrac{1}{2}$

Sum of the two zeros $= \dfrac{1}{2} + \dfrac{1}{2} = 1 = \dfrac{-(-4)}{4} = \dfrac{-(\text{Coefficient of } x)}{(\text{Coefficient of } x^2)}$

Product of two zeros $= \left(\dfrac{1}{2}\right)\left(\dfrac{1}{2}\right) = \dfrac{1}{4} = \dfrac{(\text{Constant term})}{(\text{Coefficient of } x^2)}$ **(Rest Try your self)**

Ex.3 Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$ (iii) $0, \sqrt{5}$

(iv) 1, 1 (v) $-\frac{1}{4}, \frac{1}{4}$ (vi) 4, 1

Sol. (i) Let the quadratic polynomial be $ax^2 + bx + c$

Then $-\frac{b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -1$

i,e, $\frac{b}{a} = \frac{-1}{4}$ and $\frac{c}{a} = \frac{-1}{1}$

We select a = LCM (4, 1) = 4

Then $\frac{b}{4} = \frac{-1}{4}$ and $\frac{c}{4} = -1 \Rightarrow b = -1$ and $c = -4$.

Substituting $a = 4, b = -1, c = -4$ in $ax^2 + bx + c$, we get the required polynomial

$4x^2 - x - 4$

(ii) $-\frac{b}{a} = \sqrt{2}$, $\frac{c}{a} = \frac{1}{3}$

$\Rightarrow \frac{b}{a} = \frac{-\sqrt{2}}{1}$, $\frac{c}{a} = \frac{1}{3}$

Select a = LCM (1, 3) = 3.

Then $\frac{b}{3} = -\sqrt{2}$ and $\frac{c}{3} = \frac{1}{3} \Rightarrow b = -3\sqrt{2}$ and $c = 1$.

Substituting $a = 3, b = -3\sqrt{2}$ and $c = 1$ in $ax^2 + bx + c$, we get the required polynomial

$3x^2 - 3\sqrt{2}x + 1$ **(Rest Try your self)**

Ex.4 Divide the polynomial p(x) by the polynomial g(x) and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3$, $g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5$, $g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6$, $g(x) = 2 - x^2$.

Sol. (i)
$$\begin{array}{r|l}
x^2 - 2 & x^3 - 3x^2 + 5x - 3 \;\big(\; q(x) = (x - 3) \\
 & \underset{-}{x^3} \qquad\quad \underset{+}{-2x} \\
\hline
 & -3x^2 + 7x - 3 \\
 & \underset{+}{-3x^2} \qquad \underset{-}{+6} \\
\hline
 & r(x) = (7x - 9)
\end{array}$$

Hence, Quotient q(x) = x – 3 and Remainder r(x) = 7x – 9 **(Rest Try your self)**

Ex.5 Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial.

(i) $t^2 - 3$, $2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1$, $3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1$, $x^5 - 4x^3 + x^2 + 3x + 1$

Sol. (i)

$$t^2-3\,\overline{)\,2t^4+3t^3-2t^2-9t-12\,}\,(\,q(t)=2t^2+3t+4$$

$$\begin{array}{l}
2t^4 \qquad -6t^2 \\
- \qquad\quad + \\
\hline
3t^2+4t^2-9t-12 \\
3t^2 \qquad -9t \\
- \qquad\quad + \\
\hline
4t^2-12 \\
4t^2-12 \\
- \quad + \\
\hline
\text{Remainder}=0
\end{array}$$

Hence, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$ **(Rest Try your self)**

Ex.6 Verify that $\frac{3}{2}$, – 3 and – 1 are the zeros of the cubic polynomial $p(x) = 2x^3 + 5x^2 - 6x - 9$ and then verify the relationship between the zeros and its coefficients.

Sol. $p(x) = 2x^3 + 5x^2 - 6x - 9$

Comparing the given polynomial with $ax^3 + bx^2 + cx + d$; we get

$a = 2$, $b = 5$, $c = -6$ and $d = -9$

$$p\left(\frac{3}{2}\right) = 2 \times \left(\frac{3}{2}\right)^3 + 5 \times \left(\frac{3}{2}\right)^2 - 6 \times \frac{3}{2} - 9$$

$$= 2 \times \frac{27}{8} + \frac{45}{4} - \frac{18}{2} - 9$$

$$= \frac{54+90-72-72}{8} = 0$$

and $p(-3) = 2(-3)^3 + 5(-3)^2 - (6 \times -3) - 9 = -54 + 45 + 18 - 9 = 0$

and $p(-1) = 2(-1)^3 + 5(-1)^2 - 6(-1) - 9 = -2 + 5 + 6 - 9 = 0$

Therefore, $\frac{3}{2}$, – 3 and – 1 are the zeros of $2x^3 + 5x^2 - 6x - 9$

So, $\alpha = \frac{3}{2}$, $\beta = -3$ and $\gamma = -1$

Therefore,

$$\alpha+\beta+\gamma = \frac{3}{2} + (-3) + (-1)$$

$$= \frac{3-6-2}{2} = -\frac{5}{2}$$

$$= -\frac{5}{2} = -\frac{b}{a}$$

$$\alpha\beta+\beta\gamma+\gamma\alpha = \frac{3}{2} \times (-3) + (-3) \times (-1) + (-1) \times \frac{3}{2}$$

$$= -\frac{9}{2} + 3 - \frac{3}{2} = \frac{-9+6-3}{2} = - -\frac{6}{2} = \frac{c}{a}$$

and $\alpha\beta\gamma = \frac{3}{2} \times (-3)\ (-1) = \frac{9}{2} = -\frac{d}{a}$.

Ex.7 Apply the division algorithm of find the quotient and remainder of dividing p(x) by g(x) and verify the answer, where $p(x) = x^3 - 3x^2 + 5x - 3$; $g(x) = x^2 - 2$

Sol. Dividend = $x^3 - 3x^2 + 5x - 3$

Divisor = $x^2 - 2$

$$
\begin{array}{r|l|l}
x^2 - 2 & x^3 - 3x^2 + 5x - 3 & x - 3 \\
 & \underline{\mathop{x^3}\limits_{-} \quad \mathop{-2x}\limits_{+}} & \\
 & -3x^2 + 7x - 3 & \\
 & \underline{\mathop{-3x^2}\limits_{+} \quad \mathop{+6}\limits_{-}} & \\
 & 7x - 9 &
\end{array}
$$

Stop the division here since degree of $(7x - 9) = 1$, < degree of $(x^2 - 2)$.

Quotient = $x - 3$, remainder = $7x - 9$.

Verification :

Quotient × Divisor + Remainder

$(x - 3)(x^2 - 2) + 7x - 9 = x^3 - 3x^2 - 2x + 6 + 7x - 9 = x^3 - 3x^2 + 5x - 3$

Hence, division algorithm is verified.

Ex.8 Obtain all other zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of its zeros are $\sqrt{\frac{5}{3}}$ **and** $-\sqrt{\frac{5}{3}}$.

Sol. Two of the zeros of $3x^4 + 6x^3 - 2x^2 - 10x - 5$ are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

$\Rightarrow \left(x - \sqrt{\frac{5}{3}}\right)\left(x + \sqrt{\frac{5}{3}}\right)$ is a factor of the polynomial.

i.e., $x^2 - \frac{5}{3}$ is a factor.

i.e., $(3x^2 - 5)$ is a factor of the polynomial. Then we apply the division algorithm as below :

$$
\begin{array}{r|l|l}
3x^2 - 5 & 3x^4 + 6x^3 - 2x^2 - 10x - 5 & q(x) = x^2 + 2x + 1 \\
 & \underline{\mathop{3x^4}\limits_{-} \quad \mathop{-5x^2}\limits_{+}} & \\
 & 6x^3 + 3x^2 - 10x - 5 & \\
 & \underline{\mathop{6x^3}\limits_{-} \quad \mathop{-10x}\limits_{+}} & \\
 & 3x^2 - 5 & \\
 & \underline{\mathop{3x^2}\limits_{-} \quad \mathop{-5}\limits_{+}} & \\
 & \times &
\end{array}
$$

The other two zeros will be obtained from the quadratic polynomial $q(x) = x^2 + 2x + 1$

Now $x^2 + 2x + 1 = (x + 1)^2$.

Its zeros are $-1, -1$.

Hence, all other zeros are $-1, -1$.

Ex.9 Give examples of polynomials p(x), g(x), q(x) and r(x), which satisfy the division algorithm and

(i) deg p(x) = deg q(x) (ii) deg q(x) = deg r(x) (iii) deg r(x) = 0.

Sol. (i) $p(x) = 2x^2 + 2x + 8$, $g(x) = 2x^0 = 2$; $q(x) = x^2 + x + 4$; $r(x) = 0$

(ii) $p(x) = 2x^2 + 2x + 8$; $g(x) = x^2 + x + 9$; $q(x) = 2$; $r(x) = -10$

(iii) $p(x) = x^3 + x + 5$; $g(x) = x^2 + 1$; $q(x) = x$; $r(x) = 5$.

EXERCISE – I

UNSOLVED PROBLEMS

Q.1 Find the zeros of the quadratic polynomial $x^2 + 7x + 12$, and verify the relation between the zeros and its coefficients.

Q.2 Find the zeros of the quadratic polynomial $f(x) = abx^2 + (b^2 + ac)x + bc$ and verify the relationship between the zeros and its coefficients.

Q.3 If α and β are the zeros of the quadratic polynomial $f(x) = ax^2 + bx + c$ then calculate :

(i) $\alpha^2 + \beta^2$ (ii) $\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}$

Q.4 If α and β are the zeros of the quadratic polynomial $p(s) = 3s^2 - 6s + 4$, find the value of $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} + 2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right) + 3\alpha\beta$

Q.5 If α and β are the roots (zeros) of the polynomial $f(x) = x^2 - 3x + k$ such that $\alpha - \beta = 1$, find the value of k.

Q.6 If α,β are the zeros of the polynomial $f(x) = 2x^2 + 5x + k$ satisfying the relation $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$, then find the value of k for this to be possible.

Q.7 Find a quadratic polynomial each with the given numbers as the sum and product of its zeros respectively.

(i) $\frac{1}{4}, -1$ (ii) $\sqrt{2}, \frac{1}{3}$

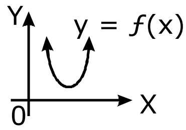
Q.8 Divide the polynomial $2x^2 + 3x + 1$ by the polynomial $x + 2$ and verify the division algorithm.

Q.9 Check whether the polynomial $t^2 - 3$ is a factor of the polynomial $2t^4 + 3t^3 - 2t^2 - 9t - 12$, by dividing the second polynomial by the first polynomial.

Q.10 Find all the zeros of $2x^4 - 3x^3 - 3x^2 + 6x - 2$, if you know that two of its zeros are $\sqrt{2}$ and $-\sqrt{2}$.

Q.11 On dividing $f(x) = x^3 - 3x^2 + x + 2$ by a polynomial g(x), the quotient and remainder were $x - 2$ and $-2x + 4$, respectively. Find g(x).

Q.12 The graph of polynomial $y = f(x)$ is shown. How many zeroes a polynomial can have ?



Q.13 Find the zeroes of the polynomial $x^2 - 16$.

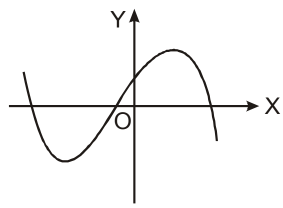
Q.14 For the polynomial $p(x) = 3x^3 - 5$. Find $p(-1)$

Q.15 Find the zeroes of a quadratic polynomial $9s^2 - 6s + 1$.

Q.16 Write the zeores of the polynomial $x^2 + 2x + 1$.

Q.17 Write the quadratic polynomial, the sum and product of whose zeroes are 3 and –2.

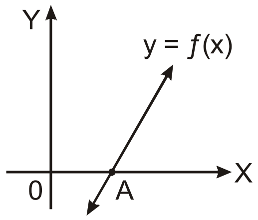
Q.18 Write the number of zeroes of the polynomial $y = f(x)$, whose graph is given as,

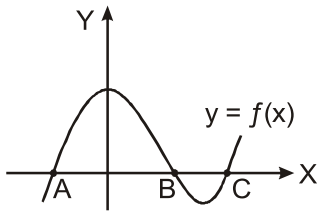


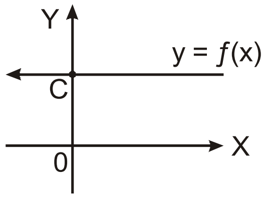
Q.19 If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a

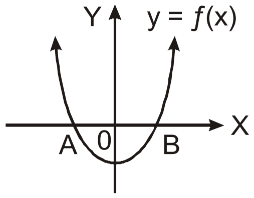
Q.20 Write the polynomial whose zeroes are $2 + \sqrt{3}$ and $2 - \sqrt{3}$.

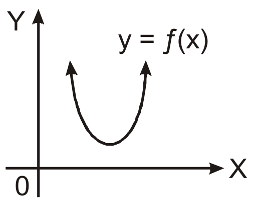
Q.21 Look at the graphs given below. Each is a graph of $y = f(x)$, where $f(x)$ is a polynomial. Which of the following corresponds to a linear or quadratic polynomial? Further for each of the graphs, find the number of zeroes of $f(x)$.

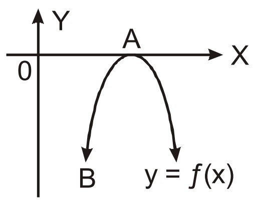
(i)


(ii)


(iii)


(iv)


(v)


(vi)


Q.22 Find the zeroes of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeroes and the coefficients of polynomial.

Q.23 Apply the division algorithm to find the quotient and remainder on dividing p(x) by q(x), where $p(x) = x^3 - 3x^2 + 5x - 3$ and $q(x) = x^2 - 2$.

Q.24 Draw the graph of the polynomial $f(x) = x^3 - x$ and hence find its zeroes from the graph.

Q.25 Find the remainder when polynomial $3x^3 + x^2 + 2x + 5$ is divided by $(x^2 + 2x + 1)$.

Q.26 Find the zeroes of the quadratic polynomial $2x^2 - 9 - 3x$ and verify the relationship between the zeores and the coefficients.

Q.27 Find all the zeroes of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of its zeroes are $\sqrt{2}$ and $-\sqrt{2}$.

Q.28 If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the value of a.

Q.29 If the product of zeores of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'.

ANSWER KEY

1. −4, −3 **2.** −b/a, −c/b

3. (i) $\frac{b^2 - 2ac}{a^2}$ (ii) $\frac{3abc - b^3}{a^2c}$

4. 8 **5.** 2 **6.** 2

7. (i) $K(x^2 - \frac{1}{4}x - 1)$ (ii) $K(x^2 - \sqrt{2}x + \frac{1}{3})$

8. $q(x) = 2x - 1$, $r(x) = 3$

9. $r(x) = 0$, yes **10.** $\sqrt{2}, -\sqrt{2}, 1, \frac{1}{2}$

11. $g(x) = x^2 - x + 1$ **12.** 0

13. ±4 **14.** −8 **15.** $\frac{1}{3}, \frac{1}{3}$

16. −1 **17.** $K(x^2 - 3x - 2)$

18. 3 **19.** 2 **20.** $x^2 - 4x + 1$

21. (i) 1 (ii) 3 (iii) No (iv) 2 (v) No (vi) 1

22. $\frac{-1}{3}, \frac{3}{2}$ **23.** $x - 3$, $7x - 9$

24. −1, 0, 1 **25.** $9x + 10$ **26.** $\frac{-3}{2}$, 3

27. $-5, \frac{3}{2}$ **28.** 3 **29.** $\frac{-3}{2}$

EXERCISE – II BOARD PROBLEMS

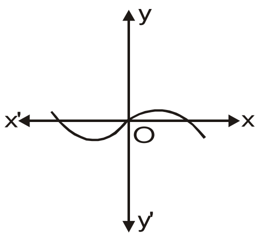
Questions Carrying 1 Mark

Q.1 Write the zeros of the polynomial $x^2 + 2x + 1$.

Q.2 Write the zeros of the polynomial, $x^2 - x - 6$.

Q.3 Write a quadratic polynomial, the sum and product of whose zeros are 3 and – 2 respectively.

Q.4 Write the number of zeros of the polynomial $y = f(x)$ whose graph is given in figure.



Q.5 If $(x + a)$ is a factor of $2x^2 + 2ax + 5x + 10$, find a

Q.6 For what value of k, (–4) is a zero of the polynomial $x^2 - x - (2k + 2)$?

Q.7 For what value of p, (–4) is a zero of the polynomial $x^2 - 2x - (7p + 3)$?

Q.8 If 1 is a zero of the polynomial $p(x) = ax^2 - 3(a - 1)x - 1$, then find the value of a.

Q.9 Write the polynomial, the product and sum of whose zeros $-\frac{9}{2}$ and $-\frac{3}{2}$ respectively

Q.10 Write the polynomial, the product and sum of whose zeros are $-\frac{13}{5}$ and $-\frac{3}{5}$ respectively.

Questions Carrying 2 Marks

Q.11 Find the zeros of the quadratic polynomial $6x^2 - 3 - 7x$ and verify the relationship between the zeros and the co-efficients of the polynomial.

Q.12 Find the zeros of the quadratic polynomial $5x^2 - 4 - 8x$ and verify the relationship between the zeros and the coefficients of the polynomial.

Q.13 Find the quadratic polynomial sum of whose zeros is 8 and their product is 12. Hence, find the zeros of the polynomial.

Q.14 If one zero of the polynomial $(a^2 + 9)x^2 + 13x + 6a$ is reciprocal of the other. Find the vlaue of 'a'

Q.15 If the product of zeros of the polynomial $ax^2 - 6x - 6$ is 4, find the value of 'a'

Q.16 Find all the zeros of the polynomial $x^4 + x^3 - 34x^2 - 4x + 120$, if two of it's zeros are 2 and – 2.

Q.17 Find all the zeros of the polynomial $2x^4 + 7x^3 - 19x^2 - 14x + 30$, if two of it's zeros are $\sqrt{2}$ and $-\sqrt{2}$

Q.18 If the polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $(ax + b)$, find a and b.

Q.19 If the polynomial $x^4 + 2x^3 + 8x^2 + 12x + 18$ is divided by another polynomial $x^2 + 5$, the remainder comes out to be $px + q$. Find the values of p and q.

Q.20 Find all the zeros of the polynomial $x^3 + 3x^2 - 2x - 6$, if two of it's zeros are $-\sqrt{2}$ and $\sqrt{2}$.

Q.21 Find all the zeros of the polynomial $2x^3 + x^2 - 6x - 3$, if two of it's zeros are $-\sqrt{3}$ and $\sqrt{3}$.

Q.22 If the polynomial $6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by polynomial $2x^2 - 5$, then find the value of a and b.

ANSWER KEY

1. –1, –1 **2.** 3, – 2 **3.** $x^2 - x - 6$ **4.** 3
5. 2 **6.** 9 **7.** 3 **8.** a = 1
9. $2x^2 + 3x - 9$ **10.** $5x^2 + 3x - 13$
11. $\left[\frac{-1}{3}, \frac{3}{2}\right]$ **12.** $\left[\frac{-2}{5}, 2\right]$
13. $x^2 - 8x + 12$; (6, 2) **14.** 3 **15.** $\frac{-3}{2}$
16. 2, – 2, – 6 and 5 **17.** $\sqrt{2}, -\sqrt{2}, -5$ and $\frac{3}{2}$
18. a = 1, b = 2 **19.** p = 2, q = 3
20. $-\sqrt{2}, \sqrt{2}$ and -3 **21.** $-\sqrt{3}, \sqrt{3}$ and $-\frac{1}{2}$
22. a = – 20, b = – 25

EXERCISE – III NTSE /OLYMPIAD /FOUNDATION PROBLEMS

Q.1 The HCF of $45(2x^4 - x^3 - x^2)$ and $75(8x^5 + x^2)$ is

(A) $15x(2x + 1)$ (B) $15x^2(2x + 1)$

(C) $15x(2x + 1)^2$ (D) $15x^2(2x + 1)^2$

Q.2 The HCF of $56(x^6y^2 - x^2y^6)$ and $72(x^5y^3 + 3x^3y^5 + 2xy^7)$ is :

(A) $8x^2y(x^2 + y^2)$ (B) $8x^2y^2(x^2 + y^2)$

(C) $8xy^2(x^2 + y^2)$ (D) $8xy^2(x + y^2)$

Q.3 HCF of $(6x^4 - 13x^3 + 6x^2)$ and $(8x^4 - 36x^3 + 54x^2 - 27x)$ is

(A) $x(2x + 3)$ (B) $x^2(2x - 3)$

(C) $x(3 - 2x)$ (D) $x(2x - 3)$

Q.4 $(x^4 + ax^3 - 7x^2 - 8x + b)$ is completely divisible by $(x^2 + 5x + 6)$, then the values of a & b are ______ .

(A) a = 2, b = 8 (B) a = 2, b = 12

(C) a = –2, b = 6 (D) none

Q.5 If $(x - k)$ is the HCF of $(3x^2 + 14x + 16)$ and $(6x^3 + 11x - 4x - 4)$, then the value of k is

(A) –2 (B) 2

(C) $\frac{2}{3}$ (D) $-\frac{1}{2}$

Q.6 If $(x + 5)$ is the HCF of $(x^2 + 2x + 3b)$ and $(6x^3 + ax^2 + 3x - 10)$, then a – 2b = _____ .

(A) 31 (B) 21

(C) 41 (D) 61

Q.7 If $(x + 1)(x + 3)$ is the HCF of $(x^2 + 3x + 2)(x^2 + 2x + a)$ and $(x^2 + 7x + 12)(x^2 + 7x + b)$, then the value of 6a + 3b = ______ .

(A) 0 (B) 36

(C) 18 (D) –36

Q.8 For the polynomials p(x) and p(x)

(A) LCM and HCF are equal

(B) LCM and HCF are not equal

(C) Cannot be determined

(D) None of these

Q.9 The LCM of $(6x^2 - x - 12)$ and $(3x^2 + x - 4)$ is

(A) $3x + 4$

(B) $(3x + 4)^2(2x - 3)(x - 1)$

(C) $(3x + 4)(2x - 3)$

(D) $(3x + 4)(2x - 3)(x - 1)$

Q.10 The LCM of $20x^2y(x^2 - y^2)$ and $35xy^2(x - y)$ is

(A) $140x^2y^2(x - y)$

(B) $140xy(x^2 - y^2)$

(C) $140x^2y^2(x^2 - y^2)$

(D) None

Q.11 Which of the following is a polynomial ?

(A) $x^2 - 6\sqrt{x} + 2$ (B) $\sqrt{x} + \frac{1}{\sqrt{x}}$

(C) $\frac{5}{x^2 - 3x + 1}$ (D) none of these

Q.12 Which of the following is not a polynomial

(A) $\sqrt{3}x^2 - 2\sqrt{3}x + 3$ (B) $\frac{3}{2}x^3 - 5x^2 - \frac{1}{\sqrt{2}}x - 1$

(C) $x + \frac{1}{x}$ (D) $5x^2 - 3x + \sqrt{2}$

Q.13 The zero of the polynomial 3x + 2 is :

(A) $-\frac{2}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

Q.14 If $p(y) = 3y^4 - 5y^3 + y^2 + 8$, then $p(-1)$ will be :

(A) 2 (B) 15

(C) 17 (D) −17

Q.15 The remainder when $p(x) = 12x^3 - 13x^2 - 5x + 7$, is divided by $(3x + 2)$ is :

(A) −1 (B) 1

(C) 2 (D) −2

Q.16 If $p(x) = 2x^2 - 3x + 5$, then $p(-1)$ is equal to :

(A) 7 (B) 8

(C) 9 (D) 10

Q.17 Zeroes of $p(x) = x^2 - 2x - 3$ are :

(A) 3 and 1 (B) 3 and −1

(C) −3 and −1 (D) 1 and −3

Q.18 Degree of polynomial $y^3 - 2y^2 - \sqrt{3}\,y + \frac{1}{2}$ is :

(A) $\frac{1}{2}$ (B) 2

(C) 3 (D) $\frac{3}{2}$

Q.19 Polynomial $2x^4 + 3x^3 - 5x^2 + 9x + 1$ is a :

(A) linear polynomial

(B) quadratic polynomial

(C) cubic polynomial

(D) biquadratic polynomial

Q.20 The polynomials $(ax^3 + 3x^2 - 3)$ and $(2x^3 - 5x + a)$ when divided by $(x - 4)$ leave the same remainder. The value of a is :

(A) 1 (B) −1

(C) 2 (D) −2

Q.21 The HCF and LCM of the polynomials P(x) and Q(x) are $(2x + 3)$ and $(2x + 3)(x + 5)(x - 4)(x - 2)(x + 1)$. If $P(x) = (2x + 3)(x^2 + x - 20)$, then $Q(x) =$

(A) $(2x + 3)(x - 4)(x + 1)$

(B) $(2x + 3)(x^2 - x - 2)$

(C) $(2x + 3)(x + 5)(x - 2)$

(D) none

Q.22 If $P(x) = (x + 2)(x^2 - 4x - 21)$, $Q(x) = (x - 7)(2x^2 + x - 6)$ and their HCF is $x^2 - 5x - 14$ then LCM of P(x) & Q(x) is _______.

(A) $(x^2 - 5x - 14)(2x^2 + 6x - 9)$

(B) $(x^2 + 5x + 6)(2x^2 - 14x + 21)$

(C) $(2x^2 + x - 6)(x^2 - 4x - 21)$

(D) $(x^2 - 5x - 14)(2x^2 + x - 6)$

Q.23 There are four polynomials P(x), Q(x), R(x) and S(x). If the HCF of each pair is $(x + 3)$ and the LCM of all the four polynomials is $(x - 1)(x - 2)(x - 3)$ then the product of four polynomials is

(A) $(x + 3)^4 (x - 1)(x - 2)(x - 3)$

(B) $(x + 3)(x - 1)(x - 2)(x - 3)$

(C) $(x + 3)(x - 1)^4 (x - 2)^4 (x - 3)^4$

(D) $(x + 3)^4 (x - 1)^4 (x - 2)^4 (x - 3)^4$

Q.24 Which of the following expressions is a polynomial?

(A) $3x^2 - 2\sqrt{5x} + 7$ (B) $\frac{x^2 - 5x + 6}{x - 3}$

(C) $\frac{x^2 - 2x + 1}{2x + 5}$ (D) None of these

Q.25 Which of the following expressions is a rational expression ?

(A) $x^3 - \sqrt{3}x^2 + \sqrt{5}x - 11$ (B) $\frac{x^2 + 3}{2\sqrt{x} - 1}$

(C) $\frac{5x^2 - \sqrt{6x} + 7}{x + 3}$ (D) $\frac{\sqrt{2}x^2 - 4\sqrt{x} + 5}{x - \sqrt{2}}$

Q.26 A rational expression whose numerator is a quadratic polynomial with zeroes, 1 and 2 and whose denominator is a monomial with zeroes 3 is _______

(A) $\frac{x^2 + x - 2}{x + 3}$ (B) $\frac{x^2 - x - 2}{x + 3}$

(C) $\frac{x^2 - x - 2}{x - 3}$ (D) $\frac{x^2 + x - 2}{x - 3}$

Q.27 Express $\frac{2x^3-54}{x^3+3x^2+9x}$ in lowest terms.

(A) $\frac{2x-6}{x}$ (B) $\frac{2(x-3)}{x^2}$

(C) $\frac{2(3-x)}{x}$ (D) None

Q.28 Express $\frac{x^4-13x^2+36}{x^3-x^2-6x}$ in lowest terms.

(A) $\frac{(x-3)(x+2)}{x}$ (B) $\frac{(x+3)(x+2)}{x}$

(C) $\frac{(x+3)(x-2)}{x}$ (D) $\frac{(x-3)(x-2)}{x}$

Q.29 Express $\frac{(x^3+y^3+z^3-3xyz)}{(x^2+y^2+z^2-xy-yz-zx)}$ in lowest terms.

(A) x + y − z (B) x + y + z

(C) x − y + z (D) x − y − z

Q.30 If $x^4 - 2x^3 + 3x^2 - mx + 5$ is exactly divisible by (x − 3), then m = _____

(A) −4 (B) −40

(C) $\frac{-40}{3}$ (D) None

Q.31 For what value of k is the polynomial $2x^4 + 3x^3 + 2kx^2 + 3x + 6$ exactly divisible by (x + 2) ?

(A) 1 (B) −1

(C) 2 (D) −2

Q.32 If α and β are the zeroes of $x^2 + 5x + 10$, then the value of (α + β) is :

(A) 5 (B) −5

(C) 8 (D) −8

Q.33 If α and β are the zeroes of $2x^2 + 5x - 10$, then the value of αβ is :

(A) $-\frac{5}{2}$ (B) 5

(C) −5 (D) $\frac{2}{5}$

Q.34 The sum and product of the zeroes of a quadratic polynomial are 2 and −15 respectively. The quadratic polynomial is :

(A) $x^2 - 2x + 15$ (B) $x^2 - 2x - 15$

(C) $x^2 + 2x - 15$ (D) $x^2 + 2x + 15$

Q.35 If one zero of the quadratic polynomial $2x^2 - 8x - m$ is $\frac{5}{2}$, then the other zero is :

(A) $\frac{2}{3}$ (B) $-\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{-15}{2}$

Q.36 If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - x - 4$, then the value of $\frac{1}{\alpha}+\frac{1}{\beta}-\alpha\beta$ is :

(A) $\frac{15}{4}$ (B) $\frac{-15}{4}$

(C) 4 (D) 15

Q.37 $a^2 + b - ab - a$ is equal to :

(A) (a − 1)(b − 1) (B) (a − 2)(b − 2)

(C) ab(a − b) (D) (a − 1)(a − b)

Q.38 a(a − 1) − b(b − 1) is equal to

(A) (a − b)(a + b − 1)

(B) (a − b)(a − b + 1)

(C) (a − b)(a + 1)

(D) none of these

Q.39 If α, β, γ are the zeroes of polynomial $2x^3 + x^2 - 13x + 6$, then the value of (αβ + βγ + γα) is :

(A) −1 (B) 1

(C) −5 (D) 30

Q.40 If α, β , γ are the zeroes of the polynomial $2x^3 + x^2 - 13x + 6$, then the value of αβγ is :

(A) 3 (B) −3

(C) $-\frac{1}{2}$ (D) $\frac{-13}{2}$

Q.41 The value of 6a + 11b, if $x^3 - 6x^2 + ax + b$ is exactly divisible by $(x^2 - 3x + 2)$ is
(A) 0 (B) 132
(C) 66 (D) −66

Q.42 Find the quadratic polynomial which is exactly divided by $(2x - 3)$ and $(x + 1)$.
(A) $2x^2 - x + 3$ (B) $2x^2 + x - 3$
(C) $2x^2 - x - 3$ (D) None

Q.43 If $f(x) = x^2 + 5x + p$ and $g(x) = x^2 + 3x + q$ have a common factor, then
$(p - q)^2$ = _____
(A) $2(5p - 3q)$ (B) $2(3p - 5q)$
(C) $3p - 5q$ (D) $5p - 3q$

Q.44 If 'a' and 'b' are unequal and $x^2 + ax + b$ and $x^2 + bx + a$ have common factor, then a + b = _______.
(A) −1 (B) 0
(C) 1 (D) 2

Q.45 If a + b + e = 0 and b + d = 0 then
$ax^4 + bx^3 + cx^2 + dx + e$ is exactly divisible by ___ .
(A) x + 1 (B) x − 1
(C) both (D) none

Q.46 $x^n + y^n$ is divisible by $(x + y)$ when 'n' is ______
(A) an even number
(B) an odd number
(C) a prime number
(D) a natural number

Q.47 $x^n - y^n$ is divisible by $(x - y)$ when "n" in ______ .
(A) an even number
(B) an odd number
(C) a prime number
(D) a natural number

Q.48 If $ax^2 + 2a^2x + b^3$ is divisible by $(x + a)$ then ____
(A) a = b
(B) $a^2 + ab + b^2 = 0$
(C) either a = b or $a^2 + ab + b^2 = 0$
(D) neither a = b nor $a^2 + ab + b^2 = 0$

Q.49 If $(x - 1)$ & $(x + 3)$ are the factors of $x^3 + 3x^2 - x - 3$ then the other factors is .
(A) x + 1 (B) x − 1
(C) x − 3 (D) x + 2

Q.50 Simplify $\frac{3x+2}{x^2-16} + \frac{x-5}{(x+4)^2}$.
(A) $\frac{4x^2+5x+28}{x^3+4x^2+16x-64}$ (B) $\frac{4x^2+5x+28}{x^3+4x^2-16x-64}$
(C) $\frac{4x^2+5x+28}{x^3-4x^2-16x-64}$ (D) $\frac{4x^2+5x+28}{x^3+4x^2-16x+64}$

Q.51 If α, β, γ be the zeroes of the polynomial p(x) such that $(\alpha + \beta + \gamma) = 3$, $\alpha\beta + \beta\gamma + \gamma\alpha = -10$ and $\alpha\beta\gamma = -24$, then p(x) is :
(A) $x^3 + 3x^2 - 10x + 24$
(B) $x^3 + 3x^2 - 10x - 24$
(C) $x^3 - 3x^2 - 10x + 24$
(D) none of these

Q.52 The value of k such that the quadratic polynomial $x^2 - (k + 6)x + 2(2k + 1)$ has sum of the zeroes as half of their product, is :
(A) 2 (B) 3
(C) −5 (D) 5

Q.53 If α, β, γ are the zeroes of the polynomial $p(x) = 4x^2 - 5x - 1$, then the value of $\alpha^2\beta + \alpha\beta^2$ is
(A) $-\frac{1}{4}$ (B) $\frac{1}{4}$
(C) $\frac{5}{16}$ (D) $-\frac{5}{16}$

Q.54 If α and β are the zeroes of the quadratic polynomial $f(t) = t^2 - 4t + 3$, then the value of $\alpha^4\beta^3 + \alpha^3\beta^4$ is :
(A) 104 (B) 108
(C) 112 (D) −112

Q.55 If zeroes of quadratic polynomial $2x^2 - 8x - m$ are $\frac{5}{2}$ and $\frac{3}{2}$ respectively, then the value of m is
(A) $-\frac{15}{2}$ (B) $\frac{15}{2}$
(C) 2 (D) 15

Q.56 If α and β are the zeroes of the quadratic polynomial $f(x) = x^2 - p(x + 1) - c$, then $(\alpha + 1)(\beta + 1)$ is equal to:

(A) $1 + c$ (B) $1 - c$

(C) $c - 1$ (D) $2 + c$

Q.57 If the polynomial $(2x^3 + ax^2 + 3x - 5)$ and $(x^3 + x^2 - 2x + a)$ leave the same remainder when divided by $(x - 2)$, then the value of a is :

(A) 2 (B) −2

(C) 3 (D) −3

Q.58 The polynomial $f(x) = x^4 - 2x^3 + 3x^2 - ax + b$ when divided by $(x - 1)$ and $(x + 1)$ leaves the remainder 5 and 19 respectively. The value of a and b is :

(A) 5 and 8 (B) 3 and 7

(C) 2 and 5 (D) 4 and 9

Q.59 If α, β are the zeroes of the polynomial $f(x) = x^2 - p(x + 1) - c$ such that $(a + 1)(b + c) = 0$, then c is equal to :

(A) 1 (B) 0

(C) −1 (D) 2

Q.60 One of the factors of $(a^2 - b^2)(c^2 - d^2) - 4\,abcd$ is :

(A) ac − bd + bc + ad

(B) ac − bd + bc − ad

(C) cannot be determined

(D) none of these

Q.61 The sum of $\left(\frac{x-1}{x+1}\right)$ and its reciprocal is

(A) $\frac{x^2-1}{x^2+1}$ (B) $\frac{x^2+1}{x^2-1}$

(C) $\frac{2(x^2+1)}{x^2-1}$ (D) none

Q.62 If $P = \frac{1+2x}{1-2x}$ and $Q = \frac{1-2x}{1+2x}$ then $\frac{P-Q}{P+Q} =$

(A) $\frac{4x}{1+4x^2}$ (B) $\frac{1+4x^2}{4x}$

(C) $-\frac{4x}{1+4x^2}$ (D) $\frac{-(1+4x^2)}{4x}$

Q.63 Express $x - \frac{1}{x}$ as a rational expression.

(A) $\frac{x-1}{x}$ (B) $\frac{1-x}{x}$

(C) $\frac{x^2-1}{x}$ (D) $\frac{1-x^2}{x}$

Q.64 What should be added to $\frac{4x}{x^2-1}$ to get $\frac{x+1}{x-1}$?

(A) $\frac{x-1}{x^2-1}$ (B) $\frac{x-1}{x+1}$

(C) $\frac{x^2+1}{x+1}$ (D) $\frac{x^2-1}{x+1}$

Q.65 What should be subtracted from $\left(\frac{2x^2+2x-7}{x^2+x-6}\right)$ to get $\left(\frac{x-1}{x+2}\right)$?

(A) $\frac{x-2}{x-3}$ (B) $\frac{x-2}{x+3}$

(C) $\frac{x+2}{x-3}$ (D) $\frac{x+2}{x+3}$

Q.66 The additive inverse of $3x - 4 + \frac{x}{2x-1}$ is

(A) $\frac{6x^2-10x+4}{2x-1}$ (B) $-3x + 4 - \frac{x}{2x-1}$

(C) $-3x + 4 + \frac{x}{2x-1}$ (D) $-3x + 4 - \frac{x}{1-2x}$

Q.67 The reciprocal of $-3 + \frac{5}{x-2}$ is

(A) $-\frac{1}{3}+\frac{x-2}{5}$ (B) $-3 + \frac{x-2}{5}$

(C) $\frac{x-2}{11-3x}$ (D) $3 - \frac{5}{x-2}$

Q.68 Simplify : $\frac{1}{1+x+x^2} - \frac{1}{1-x+x^2} + \frac{2x}{1+x^2+x^4}$

(A) 0 (B) $\frac{1}{1+x^2+x^4}$

(C) $\frac{-1}{1+x^2+x^4}$ (D) $\frac{2x-3}{1+x^2+x^4}$

Q.69 If $A = \frac{x-1}{x+1}$ then $2A - \frac{1}{2A}$ = ______

(A) $\frac{3x^2-10x-3}{2(x^2-1)}$ (B) $\frac{3x^2-10x+1}{x^2-1}$

(C) $\frac{3x^2+10x+3}{2(x^2-1)}$ (D) $\frac{3x^2-10x+3}{2(x^2-1)}$

Q.70 Simplify : $\frac{(a^2-b^2)^3+(b^2-c^2)^3+(c^2-a^2)^3}{(a-b)^3+(b-c)^3+(c-a)^3}$

(A) 3(a + b) (b + c) (c + a)

(B) 2(a + b) (b + c) (c + a)

(C) (a + b) (b + c) (c + a)

(D) 1

Q.71 If (x + 1) is a factor of $f(x) = a_0 \cdot x^n + a_1 \cdot x^{n-1} + a_2 \cdot x^{n-2} + \ldots\ldots a_n = 0$, then

(A) $a_0 + a_1 + a_2 + \ldots\ldots + a_n = 0$

(B) $a_0 + a_2 + a_4 + \ldots\ldots = 0$

(C) $a_1 + a_3 + a_5 + \ldots\ldots = 0$

(D) $a_0 + a_2 + a_4 + \ldots\ldots = a_1 + a_2 + a_5 + \ldots\ldots$

Q.72 If $P = \frac{x^3+y^3}{(x-y)^2+3xy}$, $Q = \frac{(x+y)^2-3xy}{x^3-y^3}$ and $R = \frac{xy}{x^2-y^2}$ then the value of (P ÷ Q) × R is

(A) x + y (B) xy

(C) x − y (D) none

Q.73 Simplify : $\frac{(x+y)^2-z^2}{(x+y+z)^2} \div \left\{\frac{(x-z)^2-y^2}{x^2+xy+zx} \div \frac{(x-y)^2-z^2}{x^2-xy+zx}\right\}$

(A) x + y + z (B) $\frac{1}{x+y+z}$

(C) 1 (D) None

Q.74 How much is $a^2 + 2ab + b^2$ more than $a^2 - 2ab + b^2$?

(A) 4 ab (B) −4 ab

(C) 6 ab (D) −3 ab

Q.75 The product of $x^3 + 2x^2 - 3x + 4$ and $2x^2 - 5x + 1$ is

(A) $2x^5 - x^4 - 15x^3 + 25x^2 - 23x - 4$

(B) $2x^5 - x^4 - 15x^3 - 25x^2 - 23x - 4$

(C) $2x^5 - x^4 - 15x^3 + 25x^2 - 23x + 4$

(D) $2x^5 - x^4 - 15x^3 + 23x - 4$

Q.76 If we divide $36x^2 - 12x + 1 - 49y^2$ by $6x + 7y - 1$, then the quotient is ______

(A) 6x + 1 + 7y (B) −6x − 1 − 7y

(C) 6x + 2 + 7y (D) 6x − 1 − 7y

Q.77 Find the remainder when $x^4 + 15x^3 + 6x^2 - 12x + 3$ is divided by x + 2?

(A) 52 (B) 53

(C) −52 (D) −53

Q.78 If $g(x) = x^6 + 3x^4 - 24x^2 + 3$, find g(1), g(2) and g(3).

(A) g(1) = −17, g(2) = 19, g(3) = 759

(B) g(1) = −17, g(2) = −19, g(3) = 759

(C) g(1) = −17, g(2) = −19, g(3) = −759

(D) g(1) = 17, g(2) = 19, g(3) = 758

Q.79 If $4x^4 - (a-1)x^3 + ax^2 - 6x - 1$ is divisible by 2x − 1, find the value of a.

(A) 28 (B) 29

(C) 27 (D) 26

Q.80 Find the values of "a" and "b" so that (x + 2) and (x − 1) may be factors of $x^3 + 10x^2 + ax + b$.

(A) a = 7, b = 17 (B) a = 7, b = −15

(C) a = 7, b = −18 (D) a = 7, b = −17

Q.81 The factors of $\sqrt{3}x^2 + 11x + 6\sqrt{3}$ are :

(A) $\left(x-3\sqrt{3}\right)\left(\sqrt{3}x+2\right)$

(B) $\left(x-3\sqrt{3}\right)\left(\sqrt{3}x-2\right)$

(C) $\left(x+3\sqrt{3}\right)\left(\sqrt{3}x-2\right)$

(D) $\left(x+3\sqrt{3}\right)\left(\sqrt{3}x+2\right)$

Q.82 One of the factors of $x^2 + xy^6$ is :

(A) $x^2 + y^2$ (B) x

(C) $x(x^2 + y^2)$ (D) neither A nor B

Q.83 On factorising $x^3 + x^2 + x + 1$ we get :

(A) $(x + 1)(x^2 - 1)$

(B) $(x - 1)(x^2 + 1)$

(C) $(x - 1)(x^2 - 1)$

(D) $(x + 1)(x^2 + 1)$

Q.84 On factorising $x^2 + (a + b + c)x + ab + bc$ we get:

(A) $(x + a)(x + b + c)$

(B) $(x + a)(x + b + c)$

(C) $(x + b)(x + a + c)$

(D) $(x + b)(x + b + c)$

Q.85 On factorising $x^3 - 3x^2 + 3x + 7$ we get :

(A) $(x + 1)(x^2 - 4x + 7)$

(B) $(x - 1)(x^2 + 4x + 7)$

(C) $(x - 1)(x^2 + 4x + 7)$

(D) $(x - 1)(x^2 + 4x + 7)$

Q.86 $\dfrac{0.87 \times 0.87 \times 0.87 + 0.13 \times 0.13 \times 0.13}{0.87 \times 0.87 - 0.87 \times 0.13 + 0.13 \times 0.13}$ is equal to :

(A) 0 (B) 1

(C) –1 (D) 0.1

Q.87 On factorising

$(x - y)^3 + (y - z)^3 + (z - x)^3$:

(A) $(x - y)(y - z)(z - x)$

(B) $(x - y)(y - z)(z - x)$

(C) $3(x - y)(y - z)(z - x)$

(D) none of these

Q.88 $30^3 + 20^3 - 50^3$ is equal to :

(A) 0 (B) –9000

(C) –90000 (D) 1

Q.89 If one zero of the polynomial $f(x) = (k^2 + 4)x^2 + 13x + 4k$ is reciprocal of the other, then k is equal to :

(A) 2 (B) –2

(C) 1 (D) –1

Q.90 If the sum of the zeroes of the polynomial $f(x) = 2x^3 - 3kx^2 + 4x - 5$ is 6, then the value of k is :

(A) 2 (B) 4

(C) –2 (D) –4

Q.91 $x^3 + 2x^2 + ax + b$ is exactly is divisible by $(x^2 - 1)$. Find the values of 'a' and 'b'.

(A) $a = -1, b = -2$ (B) $a = 1, b = 2$

(C) $a = -1, b = 2$ (D) $a = 1, b = -2$

Q.92 The expression $2x^3 + ax^2 + bx - 2$ leaves a remainder 7 and 0 when divided by $(2x - 3)$ and $(x + 2)$ respectively. Calculate the values of a and b.

(A) $a = 3, b = 3$ (B) $a = -3, b = 3$

(C) $a = -3, b = -3$ (D) $a = 3, b = -3$

Q.93 Find $f(4), f(-5), f(3.2)$ if $f(x) = 6.2x^2 - 4x^3 + 4.28$.

(A) $f(4) = -152.52,\ f(-5) = 659.28,$
$f(3.2) = -63.304$

(B) $f(4) = -152.52,\ f(-5) = 659.27,$
$f(3.2) = -63.304$

(C) $f(4) = -152.53,\ f(-5) = 659.28,$
$f(3.2) = 63.304$

(D) $f(4) = -152.52,\ f(-5) = -659.28,$
$f(3.2) = 63.304$

Q.94 Find the value of "k" if the expression $p(x) = kx^3 + 4x^2 + 3x - 4$ and $q(x) = x^3 - 4x + k$ leave the same remainder when divided by $(x - 3)$.

(A) $k = -2$ (B) $k = 2$

(C) $k = -1$ (D) $k = 3$

Q.95 For the expression

$f(x) = x^3 + ax^2 + bx + c$,

if $f(1) = f(2) = 0$ and $f(4) = f(0)$. Find the values of a, b & c.

(A) $a = -9, b = 20$ and $c = -12$

(B) $a = -8, b = 20$ and $c = -12$

(C) $a = 9, b = -20$ and $c = -12$

(D) $a = -8, b = -20$ and $c = 12$

Q.96 Find "a" and "b" in order that

$x^3 - 6x^2 + ax + b$ may be exactly divisible by $x^2 - 3x + 2$.

(A) $a = 11, b = -5$ (B) $a = 11, b = -6$

(C) $a = -11, b = -5$ (D) $a = 11, b = 6$

Q.97 The product of two consecutive numbers is 56. Find them.

(A) 56, –7 (B) 8, –7

(C) –8, –7 (D) –8, 7

Q.98 Find the quadratic polynomial in x, which when divided by $(x - 1)$, $(x - 2)$ and $(x - 3)$ leaves remainders of 11, 22 and 37 respectively.

(A) $2x^2 + 5x + 4$ (B) $2x^2 - 5x + 4$

(C) $3x^2 - 5x + 4$ (D) $2x^2 - 5x - 3$

Q.99 If the product of zeroes of the polynomial

$f(x) = ax^3 + 6x^2 + 11x - 6$ is 4, then a is equal to :

(A) $\frac{3}{2}$ (B) $-\frac{3}{2}$

(C) $\frac{2}{3}$ (D) $-\frac{2}{3}$

Q.100 If zeroes of the polynomial

$f(x) = x^3 - 3px^2 + qx - r$ are in A.P., then :

(A) $2p^3 = pq - r$ (B) $2p^3 = pq + r$

(C) $p^3 = pq - r$ (D) none of these

Q.101 If the zeroes of the polynomial $f(x) = k^2x^2 - 17x + k + 2$, are reciprocal of each other, then the value of k is **[NTSE 2012]**

(A) 2 (B) –1

(C) –2 (D) 1

ANSWER KEY

1.	B	**2.**	C	**3.**	D	**4.**	B
5.	A	**6.**	C	**7.**	A	**8.**	A
9.	D	**10.**	C	**11.**	D	**12.**	C
13.	A	**14.**	C	**15.**	B	**16.**	D
17.	B	**18.**	C	**19.**	D	**20.**	A
21.	B	**22.**	C	**23.**	A	**24.**	B
25.	A	**26.**	C	**27.**	A	**28.**	C
29.	B	**30.**	D	**31.**	B	**32.**	B
33.	C	**34.**	B	**35.**	C	**36.**	A
37.	D	**38.**	A	**39.**	A	**40.**	B
41.	A	**42.**	C	**43.**	B	**44.**	A
45.	C	**46.**	B	**47.**	D	**48.**	C
49.	A	**50.**	B	**51.**	C	**52.**	D
53.	D	**54.**	B	**55.**	A	**56.**	B
57.	D	**58.**	A	**59.**	A	**60.**	A
61.	C	**62.**	A	**63.**	C	**64.**	B
65.	D	**66.**	B	**67.**	C	**68.**	A
69.	D	**70.**	C	**71.**	D	**72.**	B
73.	C	**74.**	A	**75.**	C	**76.**	D
77.	D	**78.**	A	**79.**	B	**80.**	C
81.	D	**82.**	C	**83.**	D	**84.**	C
85.	A	**86.**	B	**87.**	C	**88.**	C
89.	A	**90.**	B	**91.**	A	**92.**	D
93.	A	**94.**	C	**95.**	A	**96.**	B
97.	C	**98.**	A	**99.**	A	**100.**	A
101.	A						